9th German-Polish Conference on Optimization Methods and Applications

Book of Abstracts

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Introduction

The 9th German–Polish Conference on Optimization – Methods and Applications continues a long-standing tradition of scientific collaboration between the Polish and German optimization communities. Since its inception in 1999, the conference has served as an international forum for researchers to present recent advances, exchange ideas, and establish new collaborations in optimization and its diverse applications. While maintaining its strong Polish–German identity, the conference welcomes participants from across Europe and beyond, reflecting its growing international recognition and scientific impact.

This Book of Abstracts brings together the contributions presented during the conference and its accompanying workshop. It contains abstracts of plenary lectures, invited talks, workshop presentations, and contributed papers, providing a comprehensive overview of current research in optimization, optimal control, fractional calculus, machine learning, inverse problems, mathematical modelling, and related fields. The volume reflects both the richness of contemporary optimization theory and the breadth of its applications in mathematics, engineering, computer science, and the natural sciences.

A distinctive feature of this edition is the two-day workshop held immediately prior to the conference. Intended primarily for PhD students and early-career researchers, the workshop focuses on two rapidly evolving research areas: Optimization in Machine Learning and Optimization with Fractional Calculus Methods. Led by internationally recognized experts, it offers participants an opportunity to deepen their theoretical understanding, strengthen their research skills, and foster new international scientific collaborations.

The editors hope that this volume will serve not only as a guide to the scientific programme but also as a lasting record of the conference and a valuable source of inspiration for future research and international cooperation.

The project is financed from the state budget with funds granted by the Minister of Science and Higher Education under the "Science Vectors" Programme (Project No. WNK/SP/0171/2025/01).
The Strategic Partner of the Conference is the Podlaskie Voivodeship. The Conference receives financial support from the Podlaskie Voivodeship.

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Topics

The main topics include (but are not limited to):

- theory of smooth and non-smooth optimization,
- variational problems and variational inequalities,
- optimal control,
- inverse and ill-posed problems,
- combinatorial optimization,
- vector optimization,
- solution methods and applications of optimization,
- ordinary and partial differential equations: theory and applications,
- mathematical modelling involving fractional ODEs and PDEs,
- fractional dynamic systems and applications,
- fractional signals and systems,
- integral representations for fractional differential systems,
- multi-agent systems.

Workshop

September 19–20, 2026

Objective and Scope

The two-day workshop is divided into two main thematic blocks:

- *Optimization in Machine Learning*
- *Optimization with Fractional Calculus Methods*

and is primarily designed for PhD students and other interested participants. The sessions will be conducted by invited lecturers, who are recognized experts in these fields.

The main objectives of the workshop are to deepen participants' theoretical knowledge, enhance their research competencies, and provide an inspiring environment for establishing international scientific collaboration.

Thus, the workshop offers a valuable opportunity to gain both theoretical insights and practical skills in applying state-of-the-art computational technologies in applied mathematics and engineering.

Invited Lectures

Prof. Boris Mordukhovich is a Distinguished University Professor in the Department of Mathematics and the Institute for AI and Data Science at Wayne State University, USA. An internationally recognized expert in variational analysis, optimization, systems control, and nonlinear dynamics, his research has had significant impact across mathematics, engineering, economics, and the behavioral sciences. He is a Fellow of both the Society for Industrial and Applied Mathematics (SIAM) and the American Mathematical Society (AMS), and has received multiple honorary doctorates from universities around the world. Prof. Mordukhovich has authored more than 550 publications, including books and patents, making outstanding contributions to modern mathematical optimization and its applications.

Prof. Yury Luchko is a Full Professor in the Department of Mathematics, Physics, and Chemistry at Berlin University of Applied Sciences and Technology. His research focuses on applied mathematics, particularly fractional calculus, fractional differential equations, and their applications. He earned his PhD in Mathematics from Belarusian State University in Minsk in 1993 under the supervision of Semyon Yakubovich. His work has significantly advanced the theory of fractional derivatives, including the development of operators with Sonin kernels for modelling memory-dependent processes.
He began his academic career as a postdoctoral researcher at the Free University of Berlin from 1994 to 2000. He then worked at the European University Viadrina in Frankfurt (Oder) from 2000 to 2006 before joining BHT in 2006 as a professor. He has authored or edited around 200 peer-reviewed papers and 20 books, receiving over 13,000 citations. In 2024, he received the Best Paper Award at the IFAC Conference on Fractional Differentiation and Its Applications (FDA) in Bordeaux and serves as an associate editor of *Fractional Calculus and Applied Analysis*.

Prof. Jean-Christophe Pesquet (IEEE Fellow 2012, EURASIP Fellow 2022) received the engineering degree from Supélec in 1987 and the PhD and HDR degrees from the University Paris-Sud in 1990 and 1999, respectively. From 1991 to 1999, he was a Maître de Conférences at the University Paris-Sud. From 1999 to 2016, he was a Professor with the University Paris-Est, and from 2012 to 2016, he was the Deputy Director of the CNRS Laboratoire

d'Informatique of the university.
He is currently a Distinguished Professor with CentraleSupélec, University Paris-Saclay, and the Director of the CVN Inria team. He was also a senior member of the Institut Universitaire de France from 2016 to 2021. In 2005, J.-C. Pesquet was technical chairman of ICASSP, and he was also technical chairman of ICIP 2022. He was a member of the SPTM Technical Committee from 2000 to 2005 and from 2011 to 2016 and served as an associate editor for *IEEE Signal Processing Letters* from 2004 to 2006.
He was an associate editor for *IEEE Transactions on Signal Processing* from 2009 to 2013, a senior area editor for the same journal from 2010 to 2015, and a member of the committee for the Best Paper Award of *EURASIP Journal on Advances in Signal Processing* from 2007 to 2019. He is currently an associate editor of the *SIAM Journal on Imaging Sciences*. His research interests focus on optimization methods in data science.

Prof. Dr. Marc Steinbach is Professor of Algorithmic Optimization at the Institute of Applied Mathematics, Leibniz University Hannover, a position he has held since August 2006. His research focuses on nonlinear optimization, stochastic optimization, large Karush–Kuhn–Tucker (KKT) systems, and the development of numerical algorithms. He also applies optimization techniques to practical challenges in gas and water network systems, contributing to both theoretical advances and real-world engineering applications.

Prof. Dariusz Uciński was born in Gliwice, Poland, in 1965. He studied electrical engineering at the Higher College of Engineering in Zielona Góra, Poland, from which he graduated in 1989. He received his PhD degree in 1992 and DSc degree in 2000 in automatic control and robotics from the Technical University of Wrocław, Poland. In 2007, he was awarded the title of professor, the highest academic title in Poland. He is currently a full professor at the University of Zielona Góra, Poland.
His major research activities have focused on measurement optimization for parameter estimation of systems modelled by partial differential equations. In 2005, his comprehensive monograph entitled *Optimal Measurement Methods for Distributed Parameter System Identification* was published by CRC Press in Boca Raton, Florida. He is regularly included in the Stanford/Elsevier Top 2% Scientists ranking. His other areas of expertise include optimal experimental design, optimal control, robotics, parallel computing, and machine learning.

Abstracts

Variational Analysis and Applications

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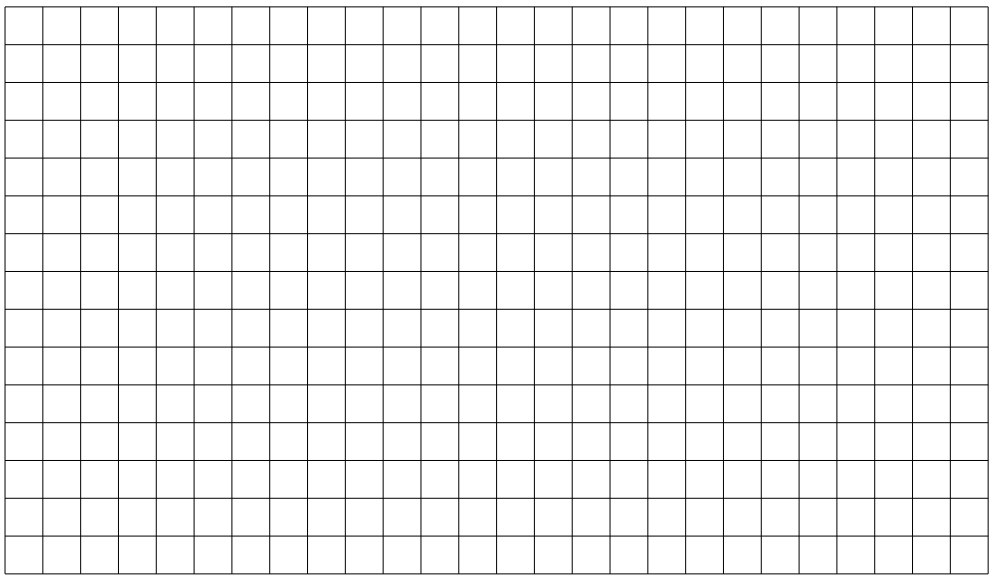
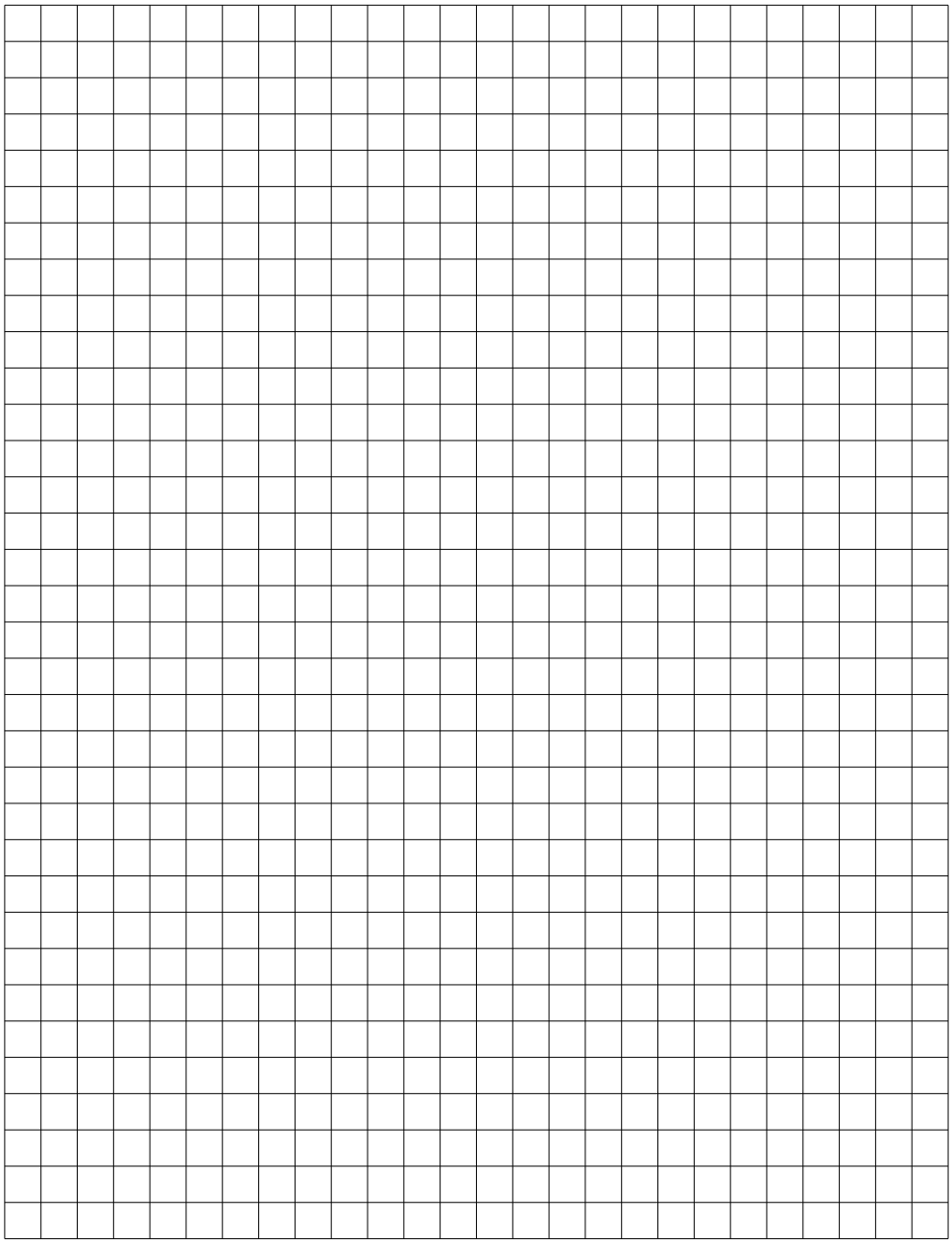
These lectures present an overview of basic principles and constructions of variational analysis and generalized differentiation with reviewing some underlying results and areas of applications. Among major applications, we list generalized Lagrange multiplier rules in nonsmooth optimization, stability and optimality conditions for control problems governed by ODEs, differential inclusions, sweeping processes, etc. Special attention will be paid to recent applications of variational analysis to sparsity optimization problems with the algorithmic developments and their implementations in practical models of proton cancer therapy.

MSC (2020): 49J53, 49J52, 49K15





On Analogues of Fermat's Theorem for Fractional Derivatives

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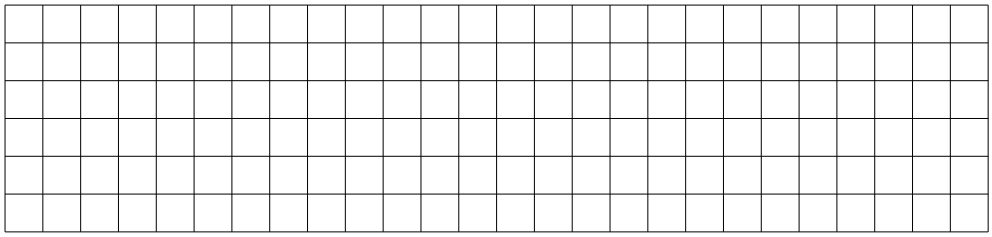
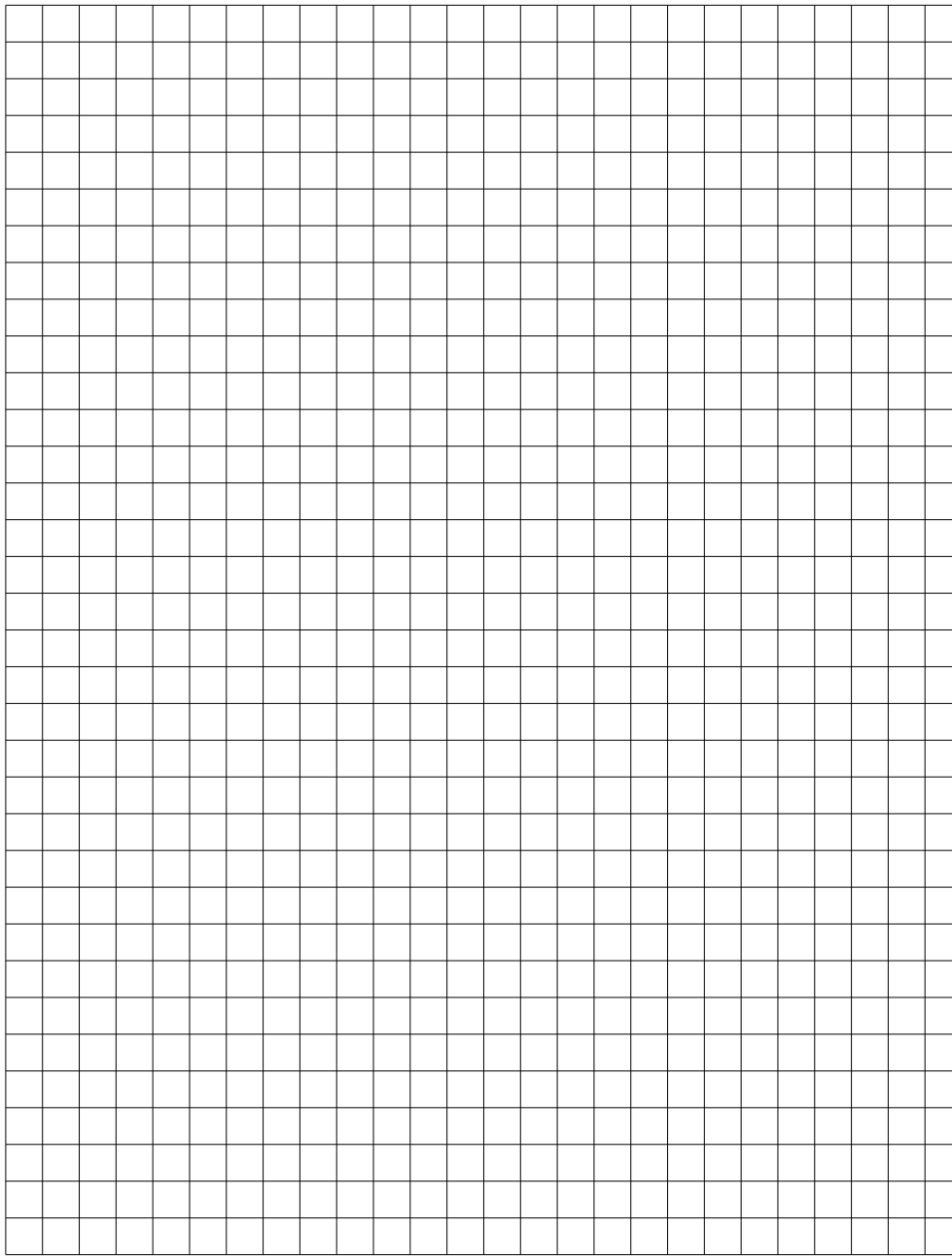
Fermat's theorem, also known as the interior extremum theorem, states that the first derivative of a differentiable function vanishes at its local extrema. Despite its simple formulation, this result plays a fundamental role in establishing many important concepts in both mathematical analysis and optimization.
In this lecture, we examine several analogues of Fermat's theorem in the context of fractional derivatives. A key observation is that, unlike in the classical case, these analogues typically appear in the form of inequalities rather than equalities. Moreover, the exact form of these inequalities depends on the particular definition of the fractional derivative being used.
Another important distinction from integer-order derivatives is that, for many fractional derivatives, their values at local extrema may be both positive and negative. An exception to this behaviour is provided by derivatives of Caputo type, which are non-negative at maximum points and non-positive at minimum points.
During the lecture, we present and discuss Fermat-type results for several classes of fractional derivatives, including the Caputo, Riemann–Liouville, and Hilfer derivatives, as well as more general fractional derivatives and their regularized forms with Sonin kernels.

MSC (2020): 26A33, 26A48





Optimization for Weakly Supervised Machine Learning

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This tutorial aims to provide a broad overview of optimization tools for machine learning, with particular emphasis on weakly supervised settings. The first part of the talk will introduce the general machine learning context and review several representative optimization techniques commonly used in this field. The focus will then shift to transductive few-shot learning, where only a limited number of labeled examples are available to classify a set of unlabeled data. We will discuss the main methodological challenges arising in this context and show how efficient optimization formulations and algorithms can be designed to address them. Finally, these concepts will be illustrated through an application.

MSC (2020): 49-01, 49M27, 62-01

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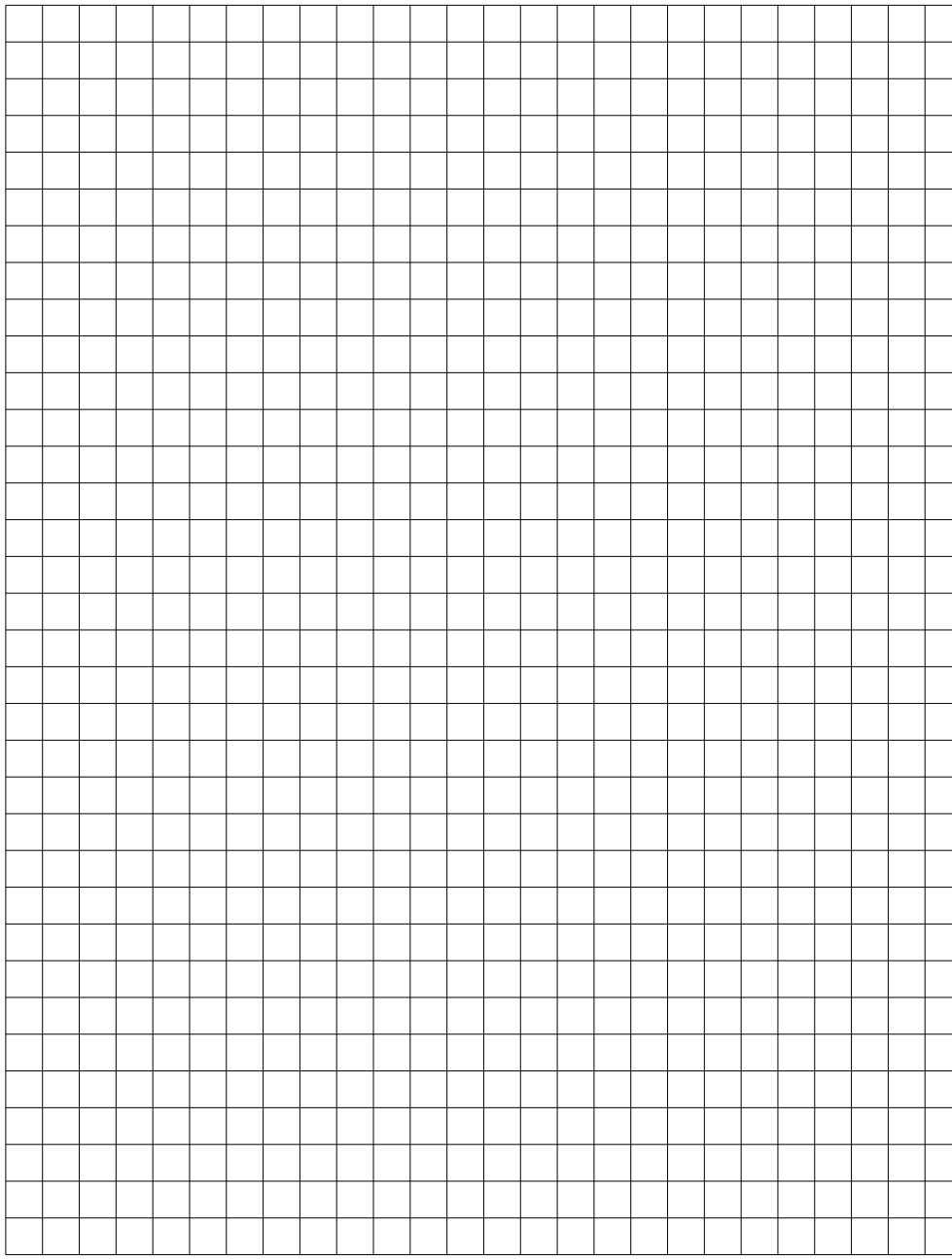

Data-Driven Elasticity in Hilbert Space

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The data-driven paradigm in elasticity replaces the classical material law by a proximity requirement with respect to a set of measured strain-stress pairs, thus converting the standard PDE boundary value problem to a discrete-continuous PDE optimization problem. We treat Dirichlet, Neumann and mixed boundary data in arbitrary spatial dimension and develop a rigorous common framework for classical and data-driven problem formulations. Our Hilbert space setting highlights the inherent problem structure, which is characterized by orthogonal decompositions of several spaces to construct a certain duality of the gradient and divergence operators. The duality arises when regarding grad and div as bounded and unbounded operators with respect to suitable norms. Considering linear kinematics, hence linear PDEs, we arrive at a universal unconstrained optimization problem for auxiliary strains and stresses whose solution determines unique physical strains, stresses and deformations and unique dual fields (Lagrange multipliers) associated to the PDE constraints. The presentation generalizes the results published in [1, 2].

MSC (2020): 35Q74, 35J57, 47N50, 49N10, 49N15

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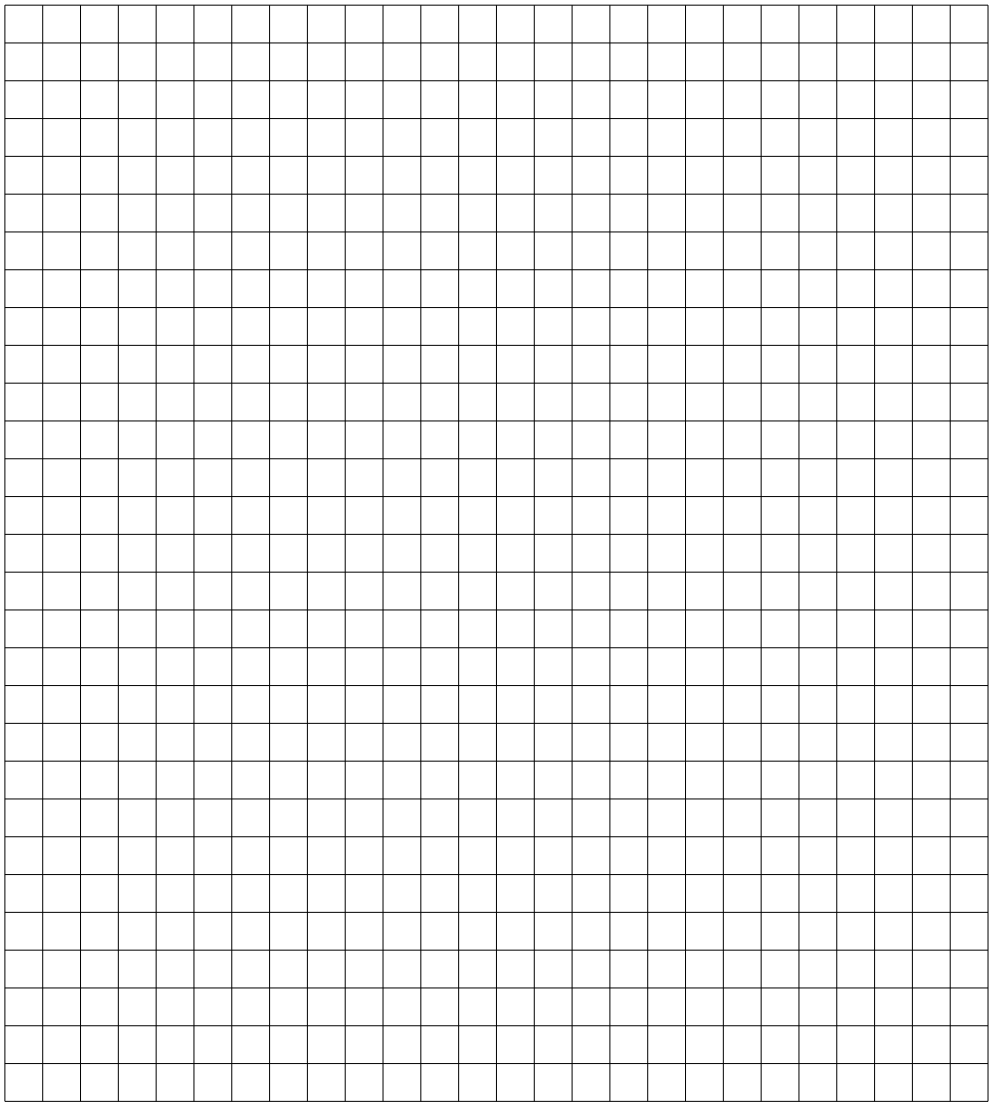

Optimal Experimental Design for Spatiotemporal System Identification

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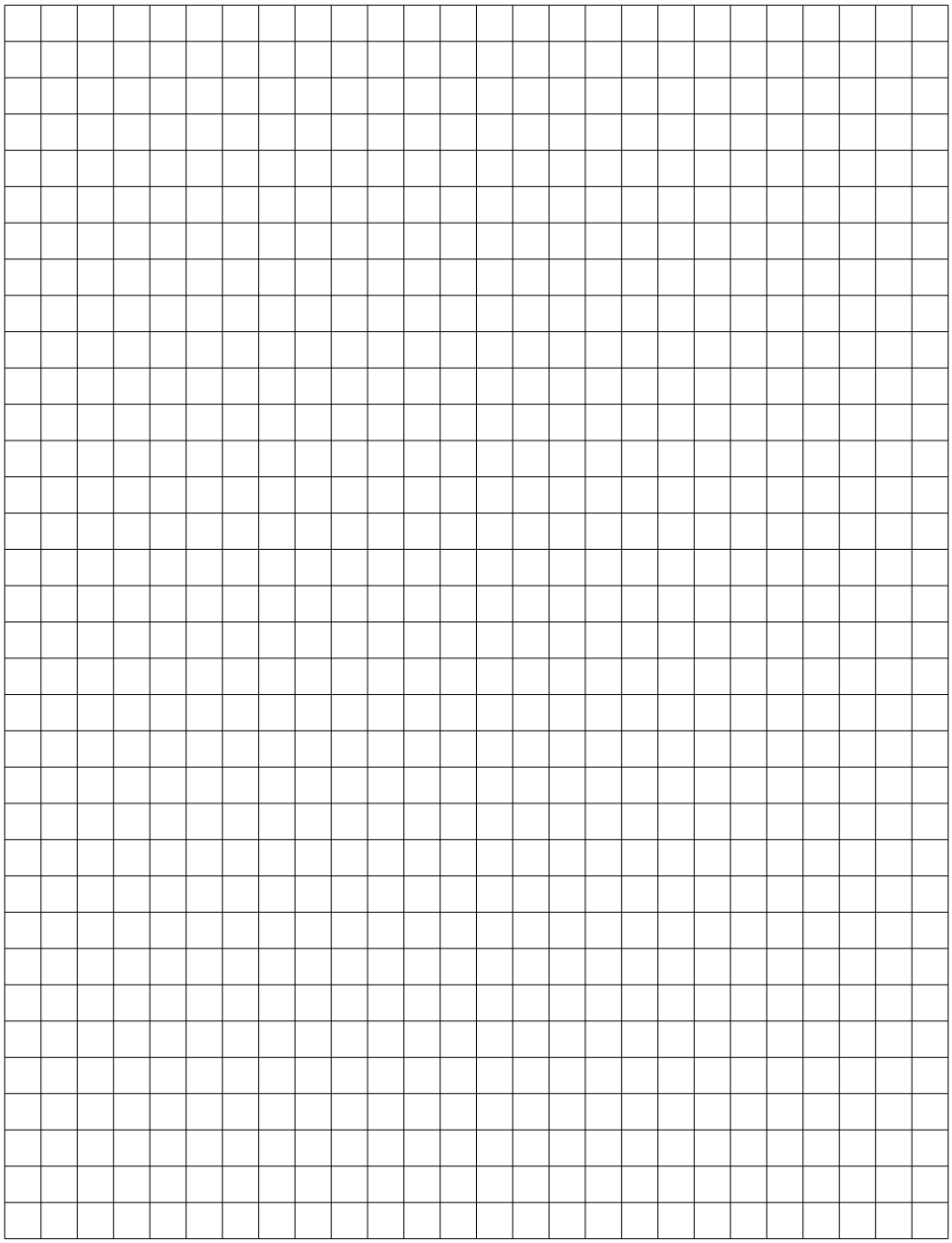
The impossibility of observing the states of distributed parameter systems over the entire spatial domain naturally raises a critical question: where should measurement sensors be placed to estimate unknown system parameters as accurately as possible? A typical example is air pollution which is modeled by a convection-diffusion equation. It is well established among researchers and practitioners that strategically positioned sensors can yield dramatic gains in parameter estimation accuracy, making efficient sensor placement strategies highly desirable. However, the inherent complexity of the sensor location problem means that few readily applicable methods exist for practical scenarios, and these remaining approaches are often underutilized.
This minicourse provides a comprehensive overview of both classical foundations and recent advances in optimal sensor placement strategies for parameter identification in dynamic distributed systems modeled by partial differential equations—an increasingly active research area. To address real-world needs, particularly in environmental monitoring, the course highlights both novel algorithms and methods adapted from related fields such as optimal control and optimum experimental design. The optimization framework primarily relies on real-valued functions of the Fisher information matrix as performance indices to be optimized with respect to sensor positions. Special emphasis is given to trajectory planning for moving and scanning sensors, as well as techniques for making optimal designs robust against parameter uncertainty. Practical applications and the strength of these methodologies are demonstrated through case studies on air quality monitoring networks. Finally, the course covers advanced topics, including optimum input design and Bayesian approaches to address the ill-posedness of the inverse parameter estimation problem.

MSC (2020): 93B30, 62K05, 35E15



Plenary Speakers

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On the Structure of Attractors for Lotka–Volterra Systems

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The talk will be devoted to the results on the structure of global attractors for the following system of Lotka–Volterra ODEs given by

$$u_i' = u_i \left(b_i + \sum_{j=1}^{n} a_{ij} u_j \right) \quad \text{for} \quad i \in \{1, \dots, n\}.$$

In autonomous case, we revisit the classical situation (cf., [6]) where the matrix $A = (a_{ij})_{i,j=1}^n$ is Volterra–Lyapunov stable. For such case, we provide results obtained in [1] and [2] and inspired by [5] on the structure of the global attractors, and their relations with the invasion graphs. We also present some results on the non-autonomous extension of the model (cf., [3] and [4]), including the existence of non-autonomous cycles.

MSC (2020): 34D45, 37M22, 37N25

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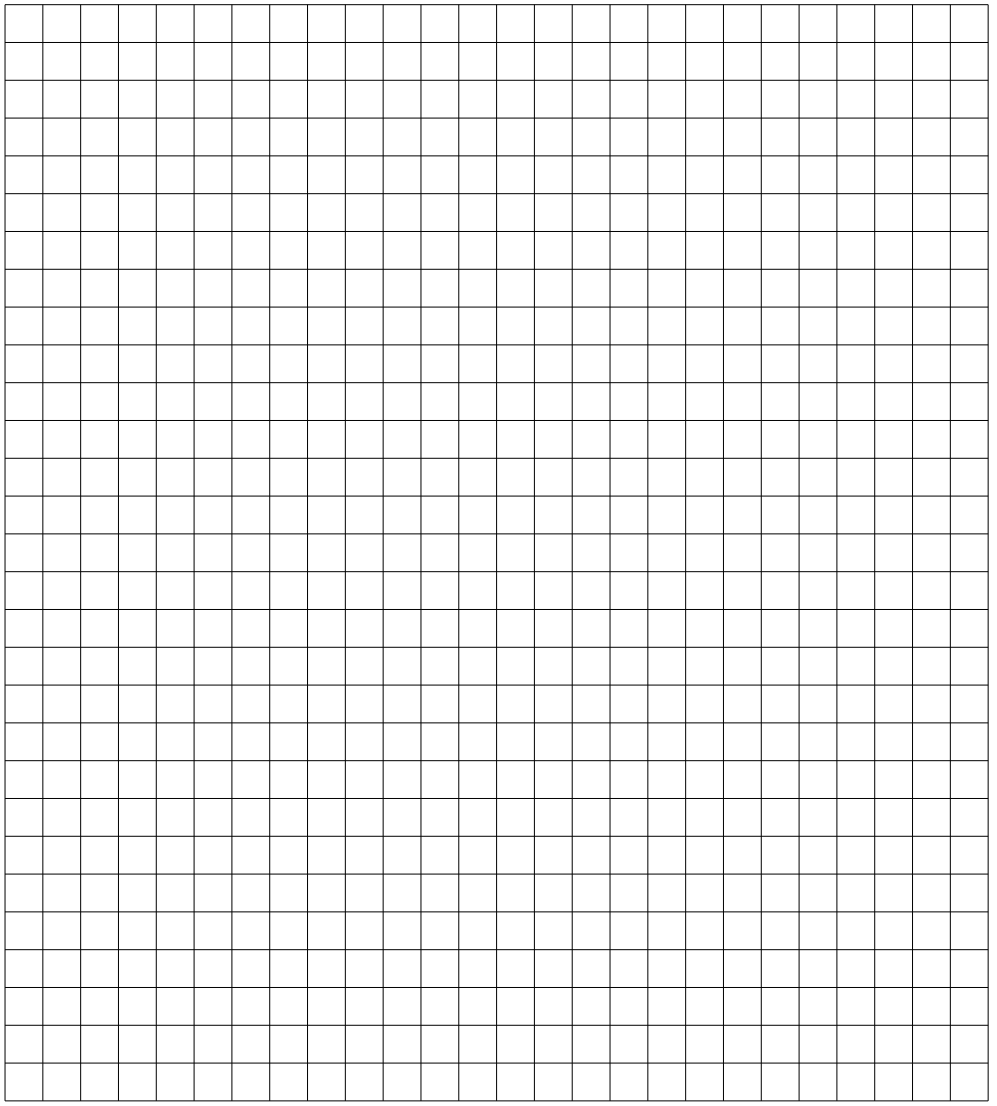

Mixed-Integer Optimal Control: From ODEs and PDEs to Graphon Dynamical Networks

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Many optimal control problems in engineering and the sciences couple discrete decisions – on/off switches, discrete actuator settings, or topology choices – with continuous dynamics governed by ordinary or partial differential equations. We survey recent algorithmic and theoretical developments for such mixed-integer optimal control problems (MIOCPs).

For systems governed by ODEs, we discuss convexification-based relaxation and rounding strategies that yield integer feasible controls with certified suboptimality bounds [1], [2], and show how such rounding schemes can be made fast and reliable enough for real-time mixed-integer nonlinear model predictive control, including recent sequential and fast-switching rounding schemes [3, 4] and their application to the supervisory control of residential building energy systems [5].

For systems governed by PDEs, we review multi-dimensional sum-up rounding and related discretization and convergence results for mixed-integer PDE-constrained optimization [6], together with recent randomized rounding schemes for elliptic control systems [7].

Finally, we discuss the extension of mixed-integer optimal control to large-scale networks of interacting dynamical systems, using the graphon limit to obtain integer control approximations and convergence guarantees that scale independently of network size [8], [9], [10].

Throughout, we highlight the convexification-and-rounding paradigm underlying these results and point out open questions concerning tightness of relaxations, real-time capability, and the treatment of switching costs and network structure.

MSC (2020): 49M25, 90C11

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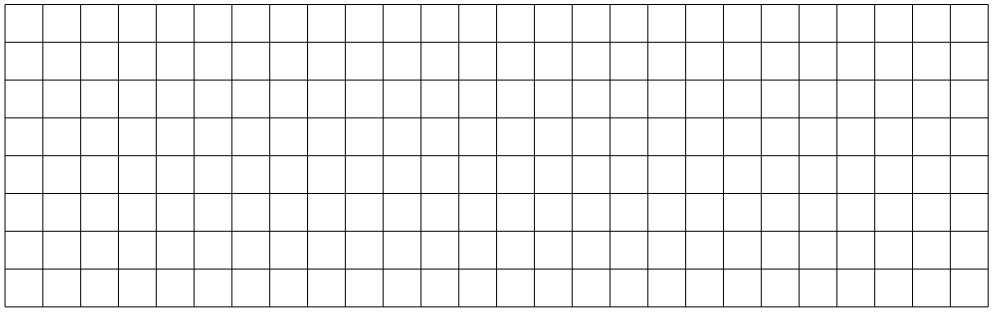

The Necessary and Sufficient Optimality Conditions for Selected Fractional Optimal Control Problems

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We will discuss the existence of optimal solutions and maximum principle for optimal control problems described by control systems with the Riemann-Liouville and Caputo derivatives and a cost functional of a Lagrange type. We consider different structures of control systems, as well as different sets of controls and we will therefore present various approaches to solve such problems. The main tools used in our study are: a theorem on the L^1-weakly lower semicontinuity of integral functionals ([4]), the lower closure theorem for orientor fields ([1]), the measurable selection theorem of Filippov type, the smooth-convex extremum principle ([3]) and the smooth extremum principle ([2]).

MSC (2020): 26A33, 49J15, 49K15

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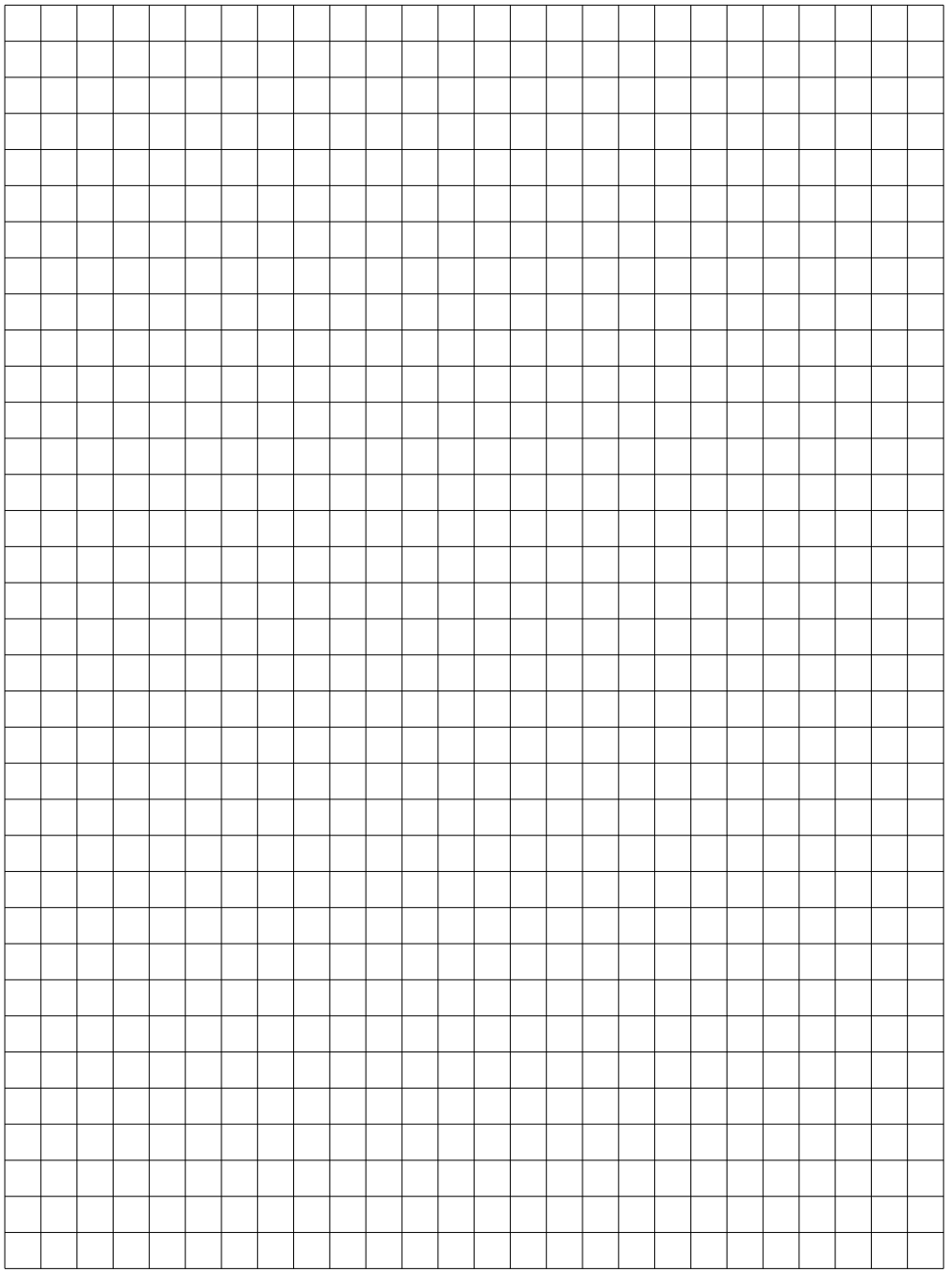

Optimizing Vascular Trees

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The synthetic generation of vascular trees aims at improving medical treatment and surgery planning or at supporting bio-printed synthetic organs. Vascular growth is believed to be governed by an energy minimization principle, leading to a discrete-continuous optimization model. We present an algorithm that combines NLP techniques for the global geometry of vascular trees with Simulated Annealing for the topology. We demonstrate that its implementation in the software SynGROW is capable of generating multiple non-intersecting full-scale trees in organs of complex shape at low computational cost. Validation against experimental results shows that the synthesized vasculature is anatomically realistic [1–3].

MSC (2020): 92C42, 90C11, 90C90

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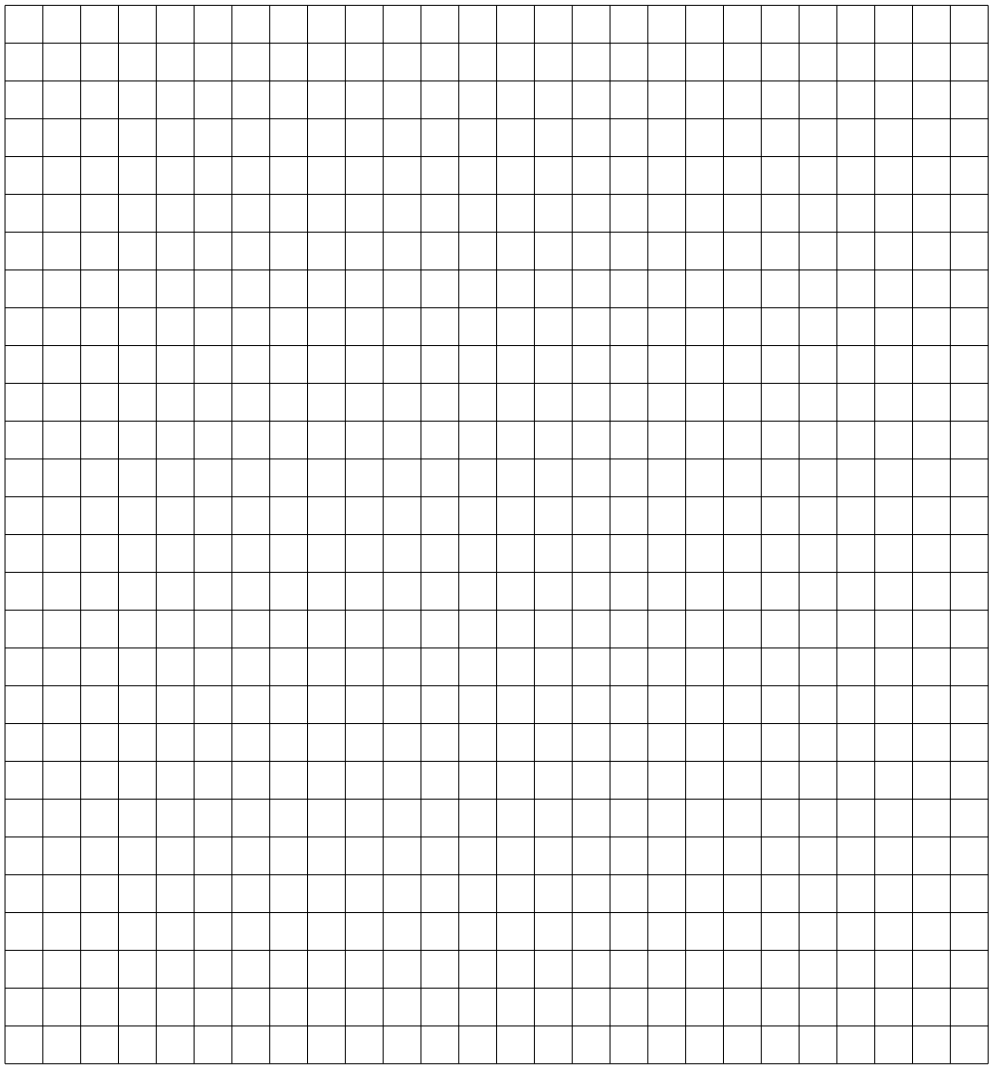
*Presenting author.



Consensus-Based Global Optimization Methods in Finite and Infinite Dimensions

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We will discuss the Consensus-based optimization methods for global optimization and sampling [1]. We will recall the underlying idea and main features of the original method for finite dimensional continuous nonconvex problems. Then we discuss analytical results on well-posedness and convergence established [2], and shed some light on variants for high-dimensional problems, multi-modal or multi-objective optimization problems. The analytical results will be illustrated by numerical results [3] highlighting the feasibility of the method. The talk will conclude with an overview of recent advances for infinite dimensional problems [4] and some outlook to current challenges.

MSC (2020): 90C26, 82C22

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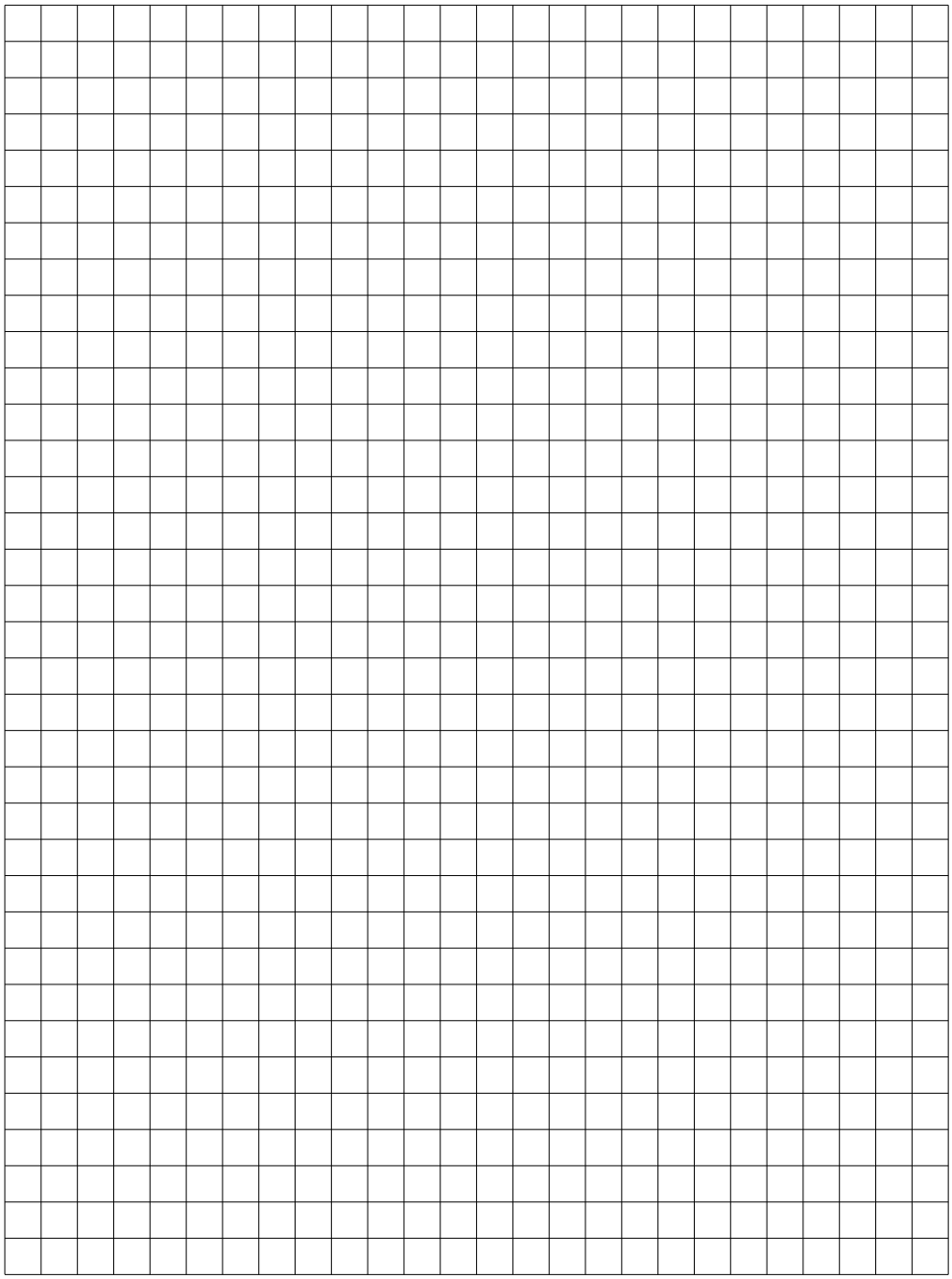

Multiple Facets of Modern Optimal Experimental Design

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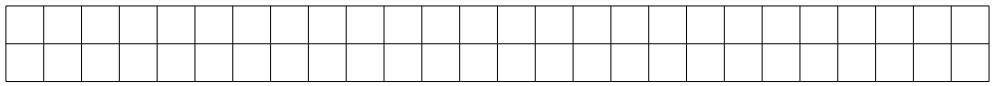
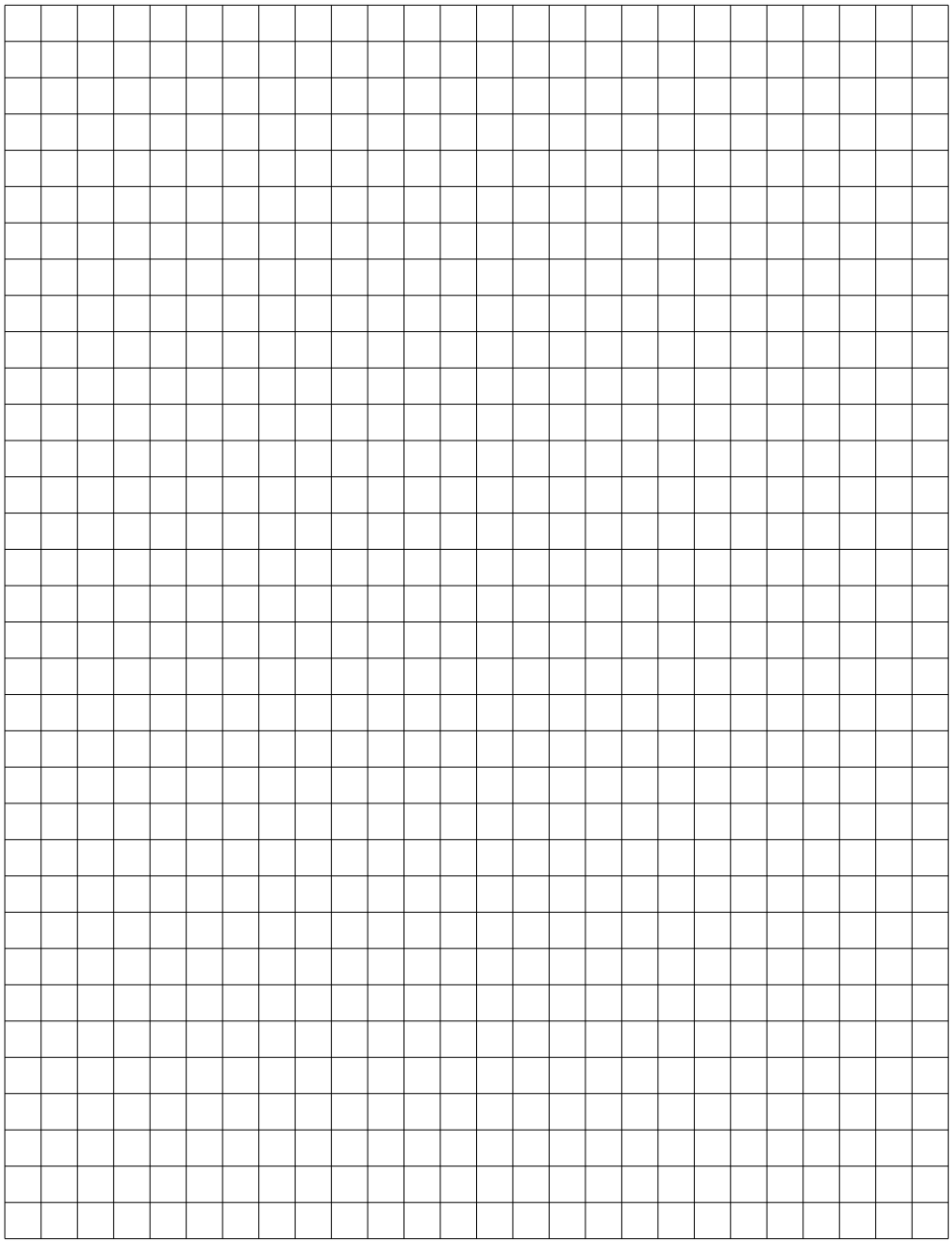
Optimum experimental design (OED) is a statistical methodology focused on capturing maximum information to build reliable predictive models across science, engineering, and industry. In parameter estimation, OED guides the configuration of controllable input variables, the strategic placement of sensors, and the efficient allocation of experimental resources. Without such systematic planning, haphazard data collection risks substantial inefficiencies—either by probing the wrong operational domains or failing to resolve critical causal effects. Although OED is broadly applicable across fields that test or refine theoretical models, its execution remains challenging due to high-dimensional parameter spaces, measurement correlations, model misspecification, parameter uncertainties, and computational expense. Furthermore, domain-specific silos often lead to fragmented terminology and redundant rediscoveries of established techniques. This lecture reviews key methodological branches of OED and examines several representative applications. Particular emphasis is placed on optimal sensor placement for dynamic systems described by partial differential equations—an increasingly vital area of research. The core optimization framework uses scalar functions of the Fisher information matrix as performance criteria to be optimized over candidate sensor trajectories and locations. Key highlights include optimal guidance strategies for mobile and scanning sensors, alongside robust techniques designed to render placement solutions independent of the unknown parameters.

MSC (2020): 93B30, 62K05, 35E15





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Principal Zeros of Mittag-Leffler Functions: Their Location and Their Role in the Construction and Analysis of Optimization Algorithms

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For the classical two-parameter Mittag-Leffler functions $E_{\alpha,\beta}$ [7] with $1 \leq \alpha, \beta \leq 2$, we investigate the following questions:

- For which values of α and β in the indicated range does $E_{\alpha,\beta}$ have real zeros?
- If $E_{\alpha,\beta}$ has at least one real zero, what can we say about the location of the real zero that is closest to the origin (sometimes called the *principal zero*)?

Answers to these questions are relevant in multiple areas, e.g. in connection with the existence of solutions to boundary value problems for certain fractional differential equations and the convergence of associated numerical solution methods [3] and with the behaviour of some algorithms for finite-time optimization [1]. The classical literature provides many results about the location of zeros of Mittag-Leffler functions that are far away from the origin (see, e.g., [7, §4.6] and the references cited therein), but investigations on zeros near the origin are scarce. In this sense, our work extends the recent findings of [2], [4], [5] and [6].

MSC (2020): 33E12, 34L15, 34L16, 65K10, 93D40

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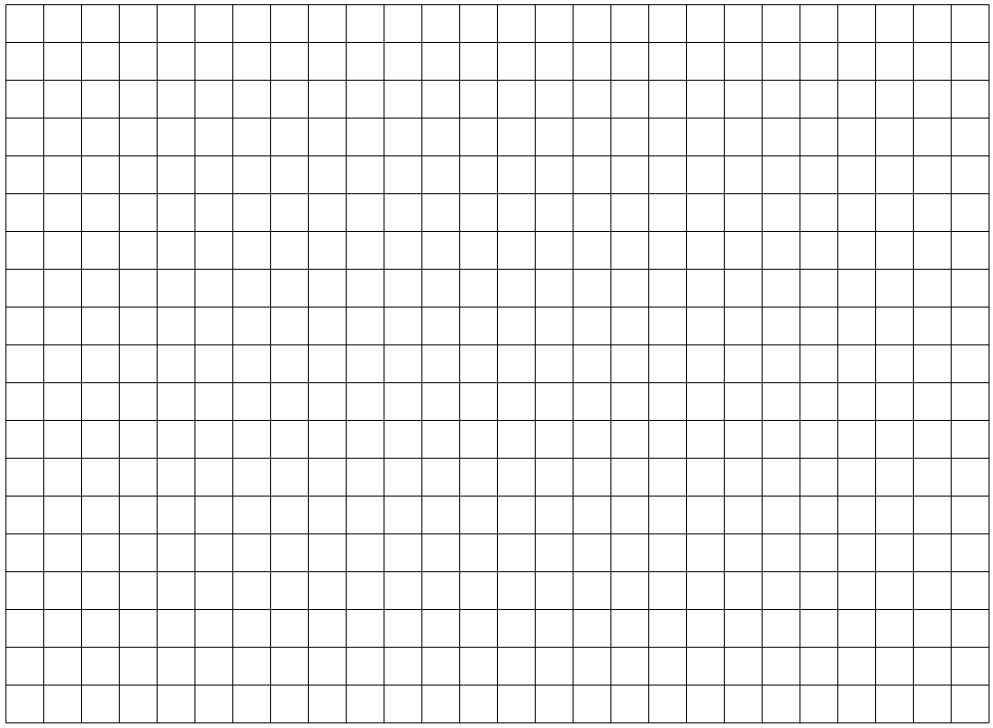

Why the Black-Scholes Model Is Good and the Gaussian Copula Is Not

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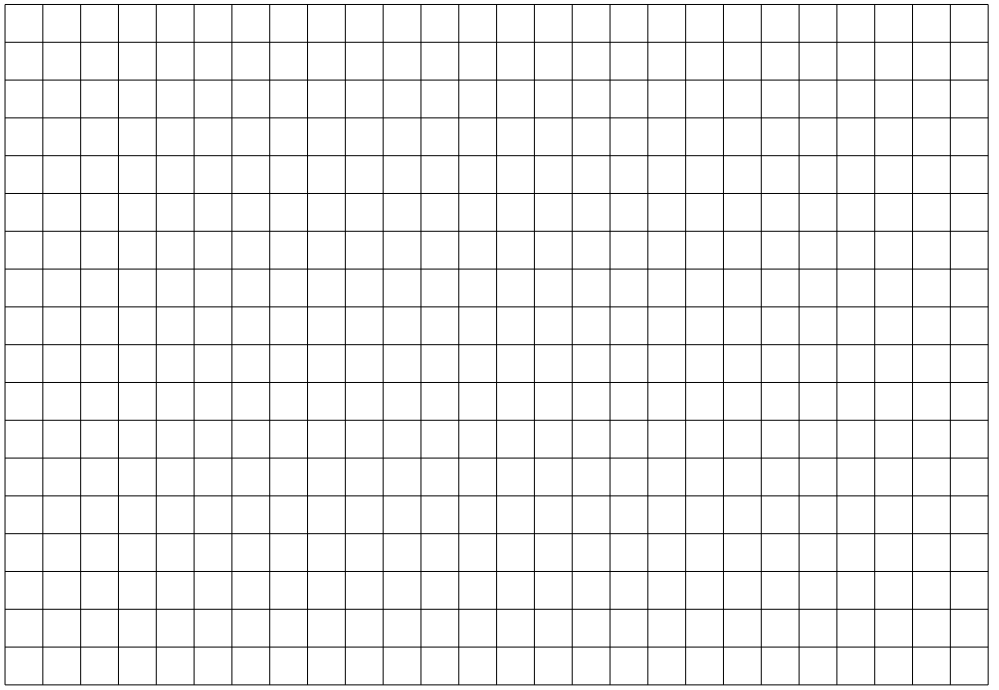
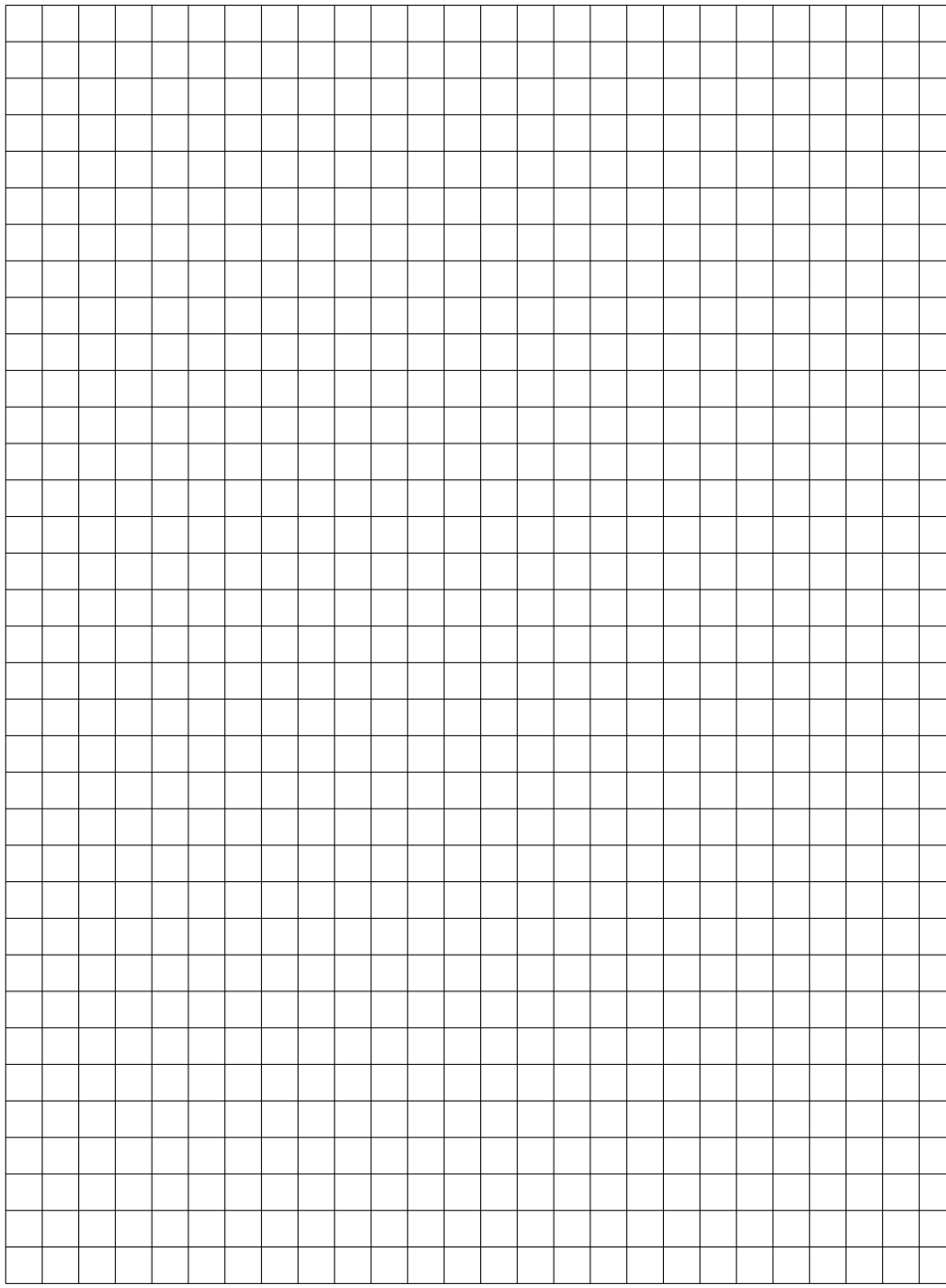
In this article, we examine why the Black-Scholes model is good and the Gaussian copula is not. In the first part of the talk, we show how the Black-Scholes model, local volatility, and stochastic volatility are mutually related. In the second part, we outline the methodology of credit portfolio analysis using the Gaussian copula and show how its shortcomings led to the credit crunch. The general conclusion is as follows: models in finance should be treated differently as in physics related sciences.

MSC (2020): 91G20, 91G40, 91G70





Is There a Formula Linking Fractional Calculus to Hamiltonian Systems ?

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The title question came up long ago for critical dynamics at phase transitions [1, Sec.V]. Subsequent research [2,3] established a link for dissipative (stochastic) systems, but for conservative (Hamiltonian) systems the question remained open.
Over the years the title question was intensely investigated by the speaker as a problem of long time limits in ergodic theory and statistical physics [4–9].
It became clear from this work that the answer to the title question is Yes. The formula is as follows:
Let the Hamiltonian H (a dimensionless energy) be given as a function $H : \Omega_N \to \mathbb{R}$ from some set Ω_N of states $\omega \in \Omega_N$ to the set $\mathbb{R}$ of real numbers. In classical mechanics $\Omega_N \cong \mathbb{R}^{6N}$ is the phase space of generalized coordinates and momenta for N particles. In non-relativistic quantum mechanics $\Omega_N \cong H^1(\mathbb{R}^{3N})$ is isomorphic to the Sobolev space $H^1(\mathbb{R}^{3N})$ and the many-body Hamiltonian H is defined as a quadratic form on Ω_N.
For fixed particle number $N \in \mathbb{N}$ the groundstate energy of the many-body system is defined as $H^0(N) = \inf_{\omega\in\Omega_N} H(\omega)$. It is well known (stability of matter) that the groundstate energy has to fulfill $-\infty < H^0(N) < \infty$ (stability of the first kind) and $H^0(N) \geq -(\text{const})\ N$ (stability of the second kind) for the thermodynamic limit $N \to \infty$ to exist. However, many pair potentials of interest violate stability of the second kind, a common violation being $H^0(N) \geq -(\text{const})\ N^2$. In this case the macroscopic ($N \to \infty$) behaviour is governed by the square root $\sqrt{H}$ of the Hamiltonian and its time evolution is expected to be governed by fractional time derivatives of order 1/2 [10]. More generally, if $H^0(N) \geq -(\text{const})\ N^\sigma$, then the desired formula is

$$\alpha = \frac{1}{\sigma}$$

where α is the power of the Hamiltonian [10].

MSC (2020): 26A33, 82B03, 82B27, 82C03

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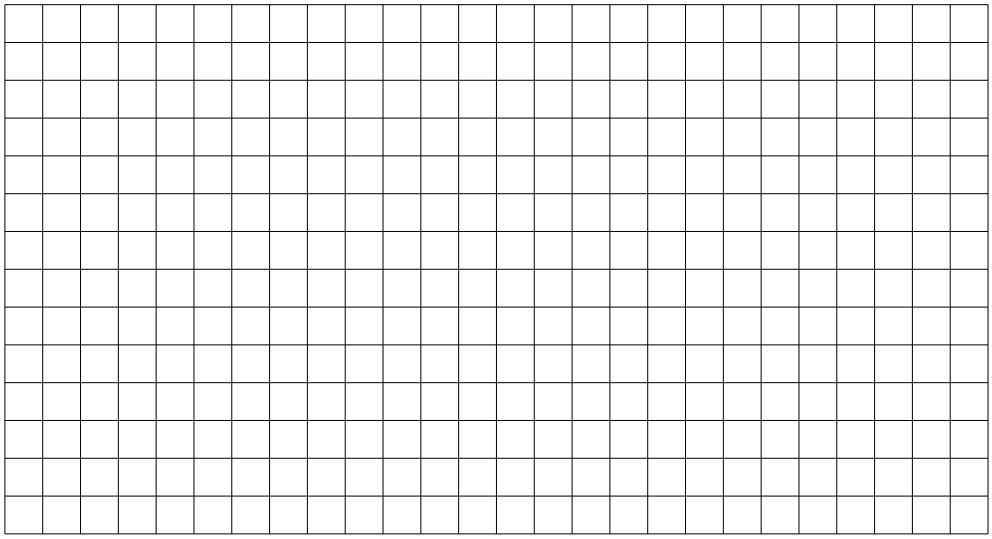

Fractional Diffusion Problems with Fixed Memory Length

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We present results on fractional diffusion problems, where fractional derivatives with full memory length were replaced by fractional derivatives with fixed memory length.
First, we solve a 1D space-fractional differential diffusion equation, where the spatial derivative is expressed by the fractional differential operator with fixed space memory length. The exact solution of the problem is constructed, taking into account the homogeneous Dirichlet boundary conditions. Additionally, since the solution is in the form of trigonometric series, we also present approximate solutions in the form of the truncated series.
Next, we study the 1D time-fractional diffusion equation with fixed memory length. Here, the time derivative is expressed by the Caputo fractional derivative with fixed memory, which leads to a solution behavior that is distinct from classical diffusion and full-memory fractional diffusion.
The time evolution and diffusion speed are discussed in detail and results on comparison between classical diffusion problem and time-fractional problems with different memory lengths are included. Additionally, we also propose an approximate solution by truncating the series and provide error estimates to control the approximation accuracy.
Finally, a 2D time-fractional diffusion problem with fixed memory length on a finite rectangular domain is discussed. Assuming homogeneous Dirichlet boundary conditions, exact weak and strong solutions are constructed using the Fourier method via 2D Laplacian eigenfunction expansions. We analyze the time evolution of the diffusion process, introducing new parameters: time-evolution coefficients, critical memory lengths, and crossing times associated with the Laplacian eigenvalues. By evaluating the L^2-norms of the solutions, the diffusion speed of the proposed model is compared to the classical diffusion equation. The results demonstrate a non-intuitive phenomenon similar to the 1D case. Namely, for specific parameter ranges, the time-fractional diffusion speed exceeds that of classical Fickian diffusion. This unexpected behavior

contrasts with the subdiffusive characteristics of full-memory fractional models, showing that a finite memory length introduces a qualitatively different transport mechanism.
The lecture is based on our latest results published in joint work [1–3].

MSC (2020): 26A33, 34A08, 35R11, 60G22

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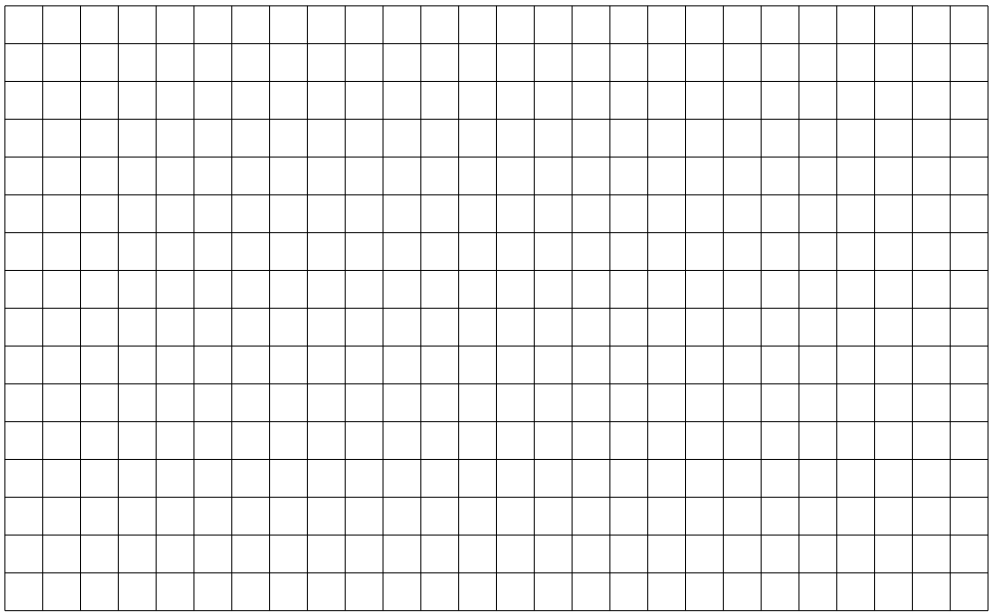

Optimization without (Exact) Adjoints

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Whenever the optimization variable appears as input to a linear map with the objective function or within a constraint, the optimality condition will also contain the adjoint in one way or another. However, there are situations in which no implementation of the adjoint is available (e.g. if the linear operator is only available via precompiled code). In this talk we will discuss what can be done in these situations. We will treat some questions of optimization with approximate adjoint like in [1] and [2] but also deal with basic question on how to compute basic quantities like the operator norm in these situations as discussed in [3] and [4].

MSC (2020): 65F35, 15A60

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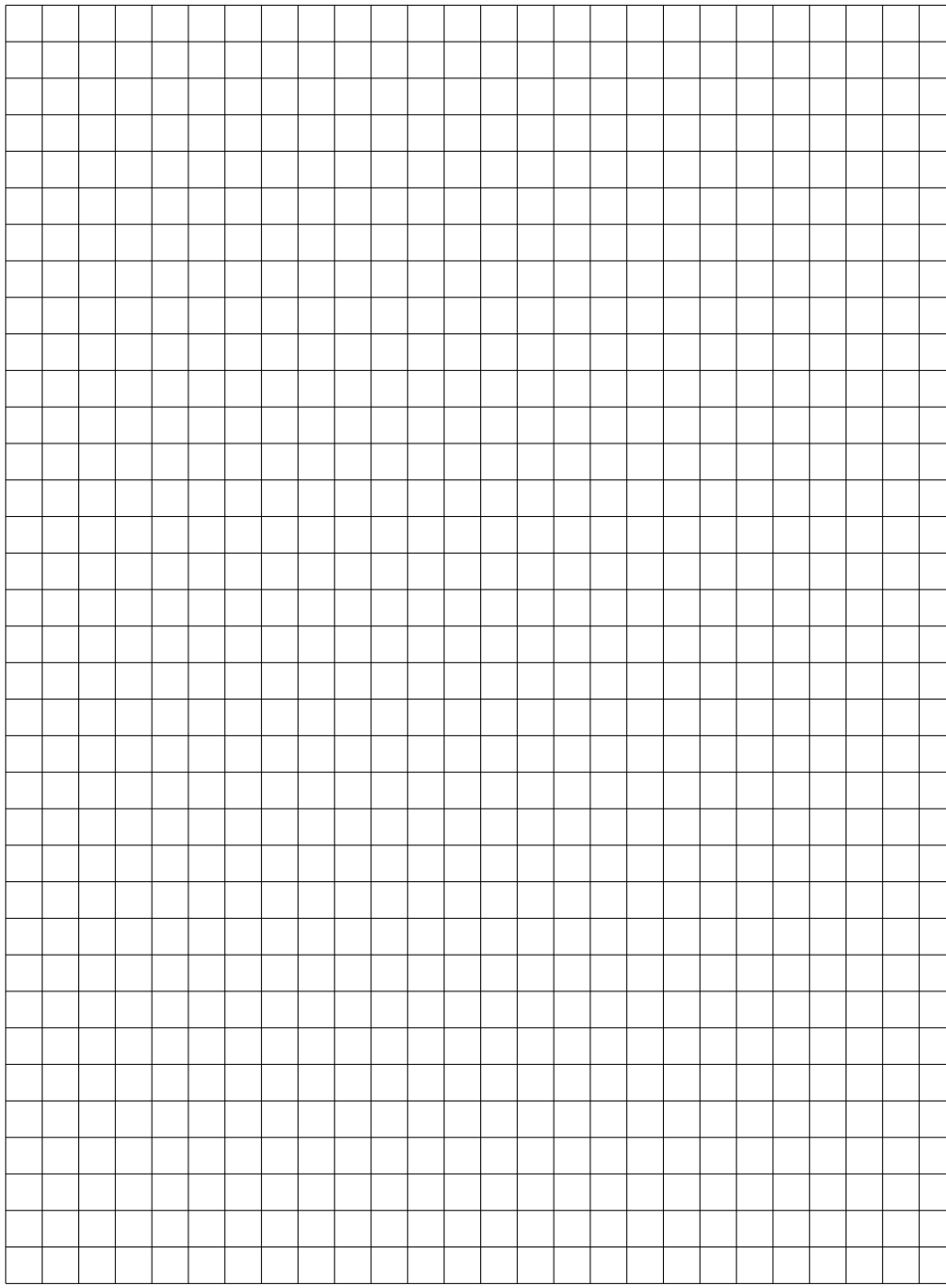

Maximum Principle for the Fractional Diffusion Equations and its Applications

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The maximum principle is a fundamental tool in the theory of elliptic and parabolic partial differential equations that plays an important role in the study of both direct and inverse problems for these equations. Fractional diffusion equations with a time-fractional derivative of order between zero and one can be viewed as interpolating between elliptic and parabolic PDEs. Therefore, it is natural to expect that the maximum principle also holds for such equations. In this talk, we present some results concerning the maximum principle for the time-fractional diffusion equations derived in [1–3], and other related publications. In particular, we discuss the maximum principle for the fractional diffusion equation with the general fractional derivative and the positivity property of solutions to the fractional diffusion equation with the Caputo derivative in the fractional Sobolev spaces. Finally, we address the maximum principle for an abstract space- and time-fractional evolution equation formulated in a Hilbert space setting.

MSC (2020): 26A33, 33E12, 35B45, 35B50, 35K99, 45K05

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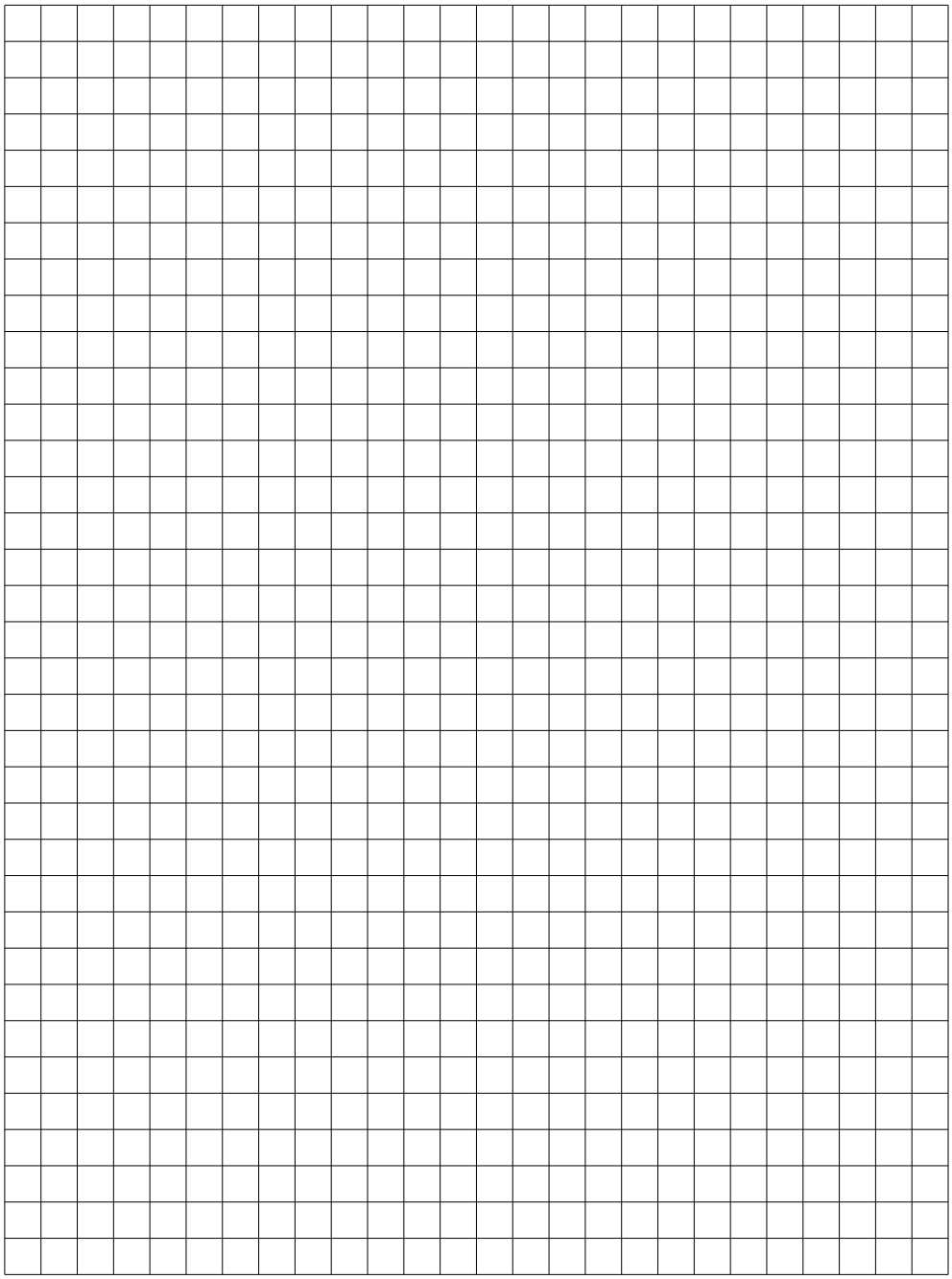

Current Achievements of Fractional Mechanics

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The mechanical behaviour of structural elements at reduced length scales frequently departs from the predictions of classical continuum mechanics, giving rise to pronounced size-dependent, non-local phenomena that cannot be captured by local constitutive formulations. This contribution presents a coherent overview of recent achievements in the modelling, experimental identification, and optimisation of scale-sensitive structures within the space-Fractional Continuum Mechanics (s-FCM) approach, developed at the Division of Computer Aided Design, Poznań University of Technology.

The theoretical foundations are first outlined, wherein non-locality is introduced through fractional-order deformation gradients governed by two material parameters: the length scale, ℓ_f, and the fractional order, α. On this basis, space-Fractional Beam and Plate models are formulated using fractional variational calculus, and their governing equations are solved numerically. Model parameters are subsequently identified and validated against experimental data obtained for polymer micro-cantilevers and composite microplates, confirming the transition between local and non-local regimes as the ratio between the internal length scale and the characteristic structural dimension varies.

The second part of the presentation addresses optimisation problems arising in scale-sensitive systems, including the identification of discrete microstructures reproducing the static and dynamic response of fractional continua [1], the design of prescribed dynamic spectral characteristics through length-scale modification [2], and damage identification in beam-like nano-joints, where

both static and dynamic sensitivity analyses are compared using experimental data obtained for silver nanowires [3].
Collectively, the results demonstrate that the s-FCM framework provides a physically consistent and computationally tractable tool for the modelling, parameter identification, and optimisation of engineering structures in which size effects play a governing role, and constitute a representative overview of current achievements in fractional mechanics.

MSC (2020): 26A33, 74A60, 74G75, 74S20, 74S05

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Contributed Talks

Global Solutions for the Sensors Placement Problem via Weakly Convex Optimization

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We will discuss recently obtained results based on our latest work [1]. We address the problem of optimally placing a limited number of sensors to reconstruct high-dimensional signals without knowledge of the underlying dynamics. The task is formulated as a nonconvex combinatorial optimisation problem and recast as a weakly convex constrained projection problem. This reformulation allows us to compute ε-global solutions using the Inexact Cutting Sphere algorithm. We further propose the Inverse Cutting Sphere algorithm, which starts from any feasible heuristic solution and either improves it by a prescribed tolerance ε or certifies its ε-global optimality. The framework is evaluated on pressure reconstruction for NACA airfoils using XFOIL data.

MSC (2020): 90C26, 90C30, 93B07

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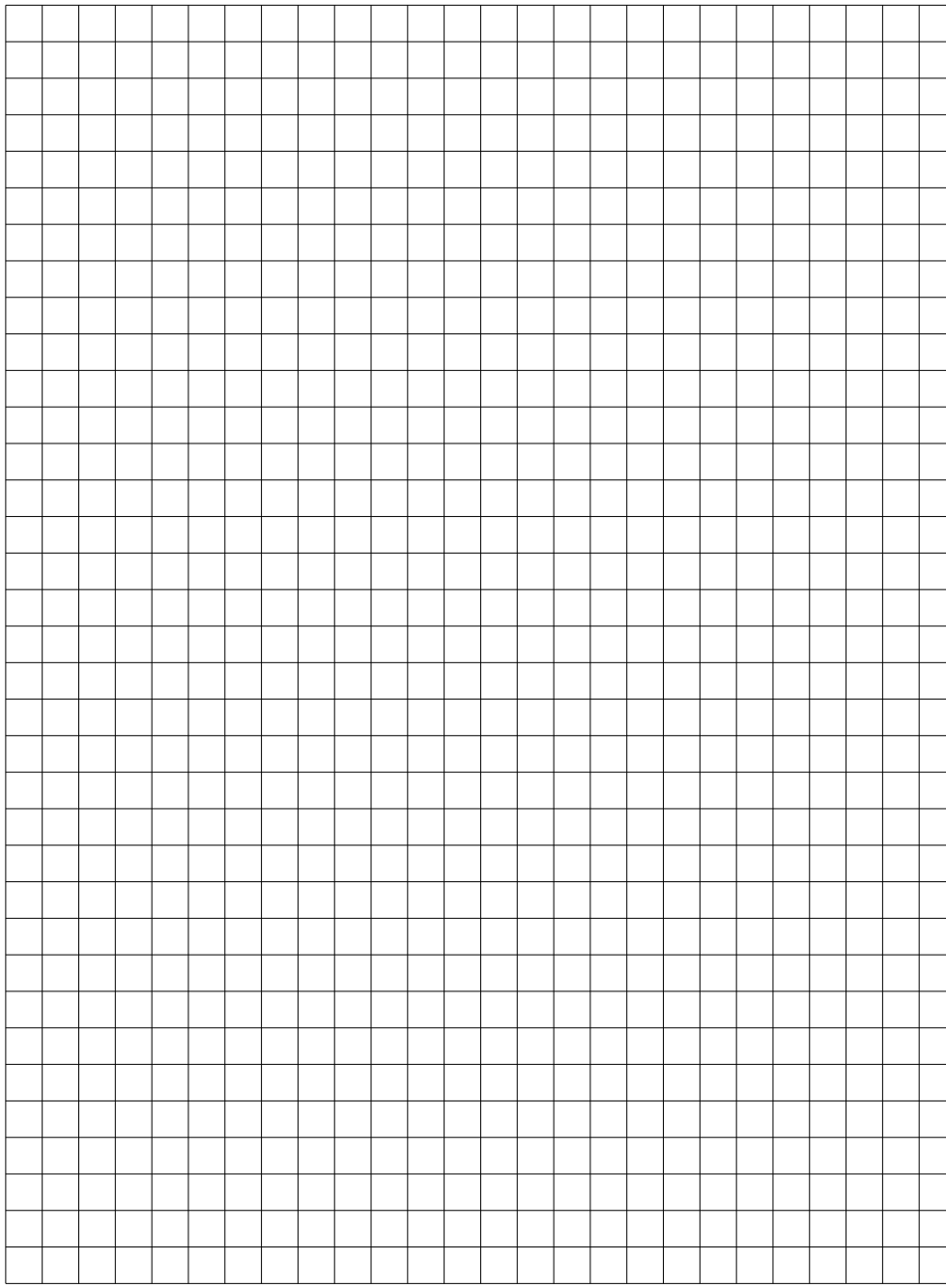

Randomized Kaczmarz Algorithm in the Product Space

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The history of the Kaczmarz method spans nearly 90 years. In 1937, the Polish mathematician Stefan Kaczmarz published a three-page paper presenting an iterative method for solving systems of linear equations. The method relies on cyclic projections onto the hyperplanes corresponding to the individual equations of the system. Kaczmarz demonstrated that if the system's matrix is non-singular, the sequence generated by the method converges to the solution. Over the past 90 years, the method has undergone numerous modifications and generalizations, finding applications in a wide range of practical fields. In my presentation, I will outline the history of generalizations of this method. I will also discuss the so-called randomized Kaczmarz method, proposed by Strohmer and Vershynin in 2009 and generalized by Zouzias and Freris in 2013, and present some new results concerning this method.

MSC (2020): 65F10, 65F20, 65F50

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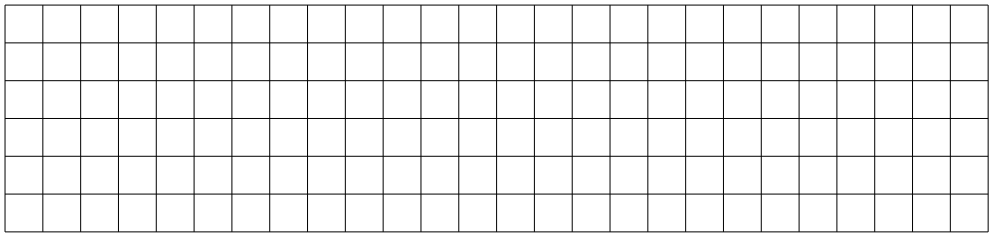

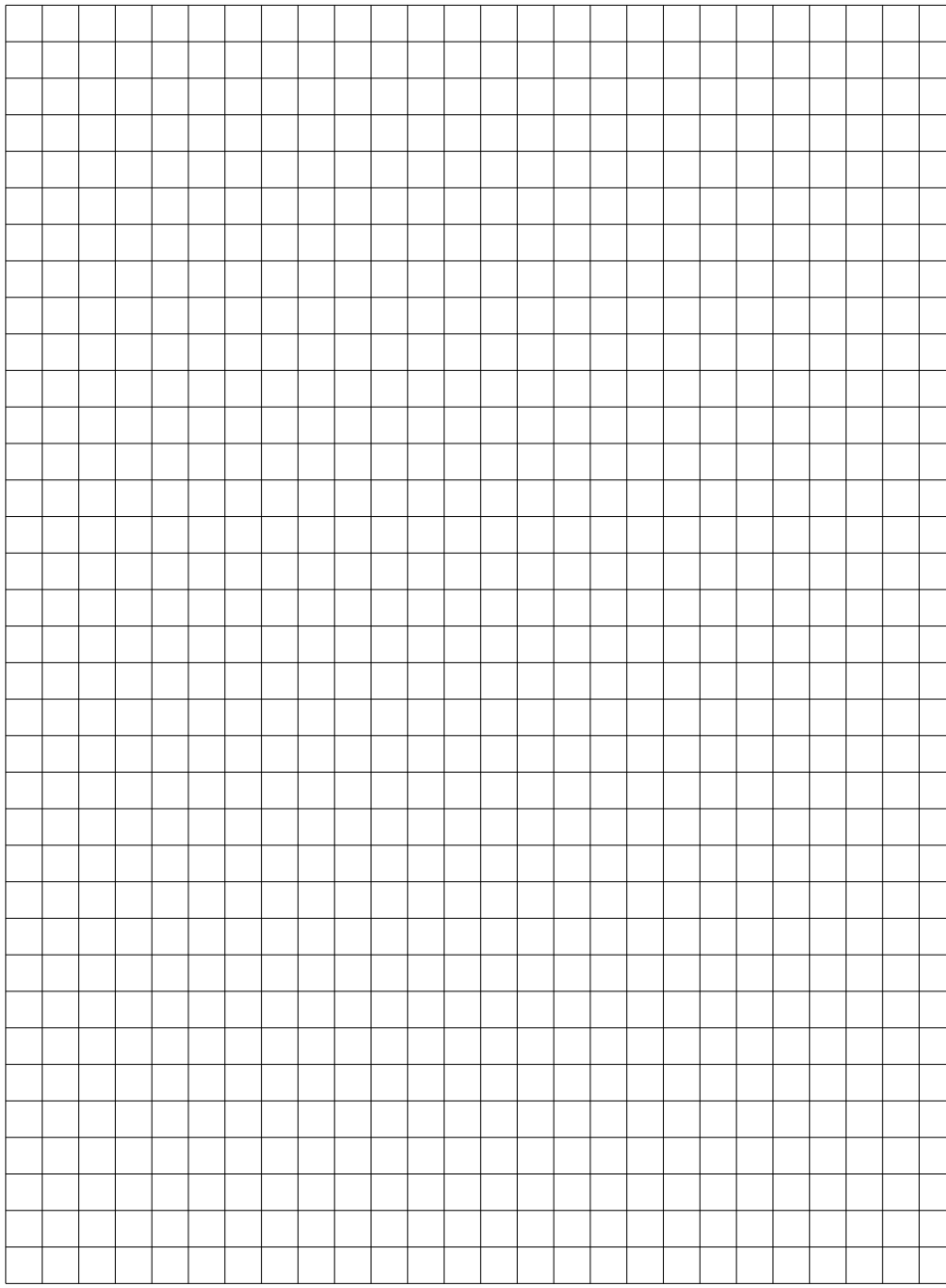

A Discretized Stackelberg Approach to Bilevel Optimization for Decarbonizing Urban Heating Systems

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Decarbonizing residential heating is critical to achieving climate neutrality in Germany and across Europe [1]. This transition requires large-scale adoption of low-carbon technologies such as heat pumps, renewable electricity, and district heating systems. Most energy system models have been formulated as a single-level mixed-integer linear program (MILP) that minimizes total system cost from a central planner's perspective [2]. In practice, however, investment decisions are made by individual households responding to retail energy prices, creating a hierarchical optimization structure: a provider sets tariffs (upper level) and households independently minimize their private costs (lower level) [3]. This interaction constitutes bilevel formulation.

This paper presents a discretized decomposition approach to solve this bilevel problem for residential heating system investment planning [4]. The lower-level household problems are formulated as independent MILPs with piecewise-linear investment cost functions, binary technology selection, and hourly operational dispatch. Each household MILP is solved for a discrete grid of retail tariff combinations, constructing a mapping from price vectors to optimal demand responses. The upper-level provider problem then optimizes over this discrete response surface to identify tariff configurations that maximize profit or minimize system-wide welfare loss. A centralized single-level MILP serves as a benchmark, and the Price of Anarchy quantifies the efficiency loss attributable to incentive misalignment.

The framework is applied to representative residential building clusters in Schweinfurt, Germany. Sensitivity analysis across a structured tariff grid is conducted to identify technology switching boundaries and quantify the welfare loss compared to the centralized benchmark. The results show that energy price structures strongly influence both technology adoption and decarbonization outcomes. Compared to centralized optimization, decentralized decision-making leads to higher total system costs but can improve household economic outcomes, and, in several scenarios, accelerate electrification. This reflects households' direct response to favourable energy prices and technology costs, even when these decisions are not system-optimal.

MSC (2020): 90C11, 91A65, 90C90

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*Presenting author.

Planning Tourist Trips Using Genetic Algorithms

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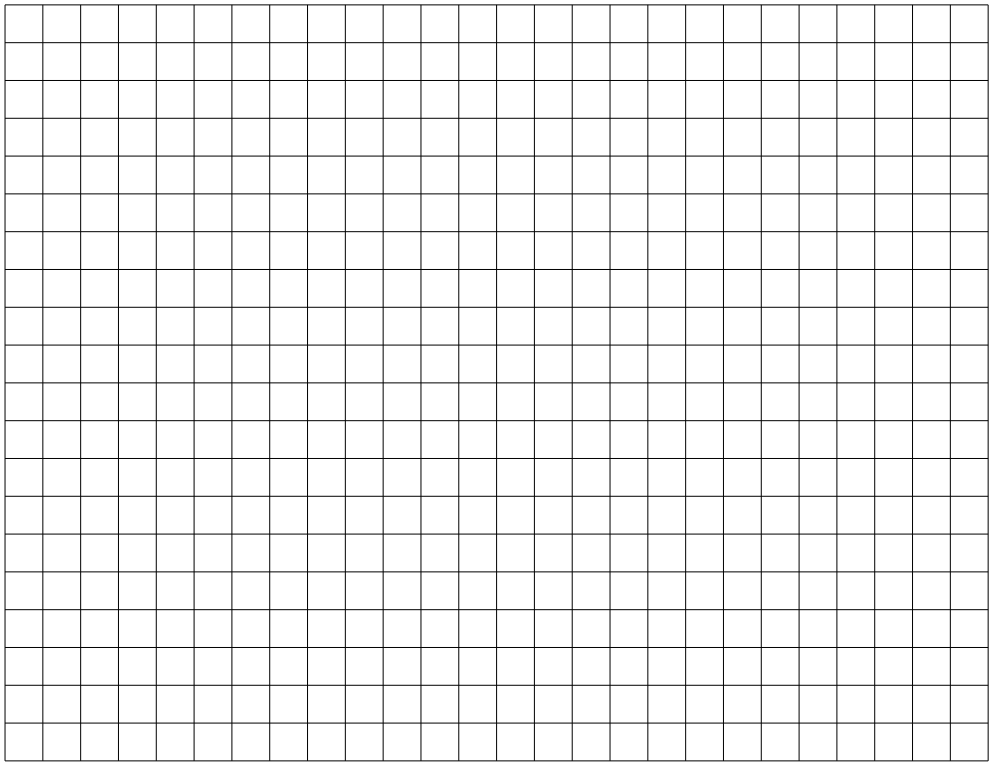
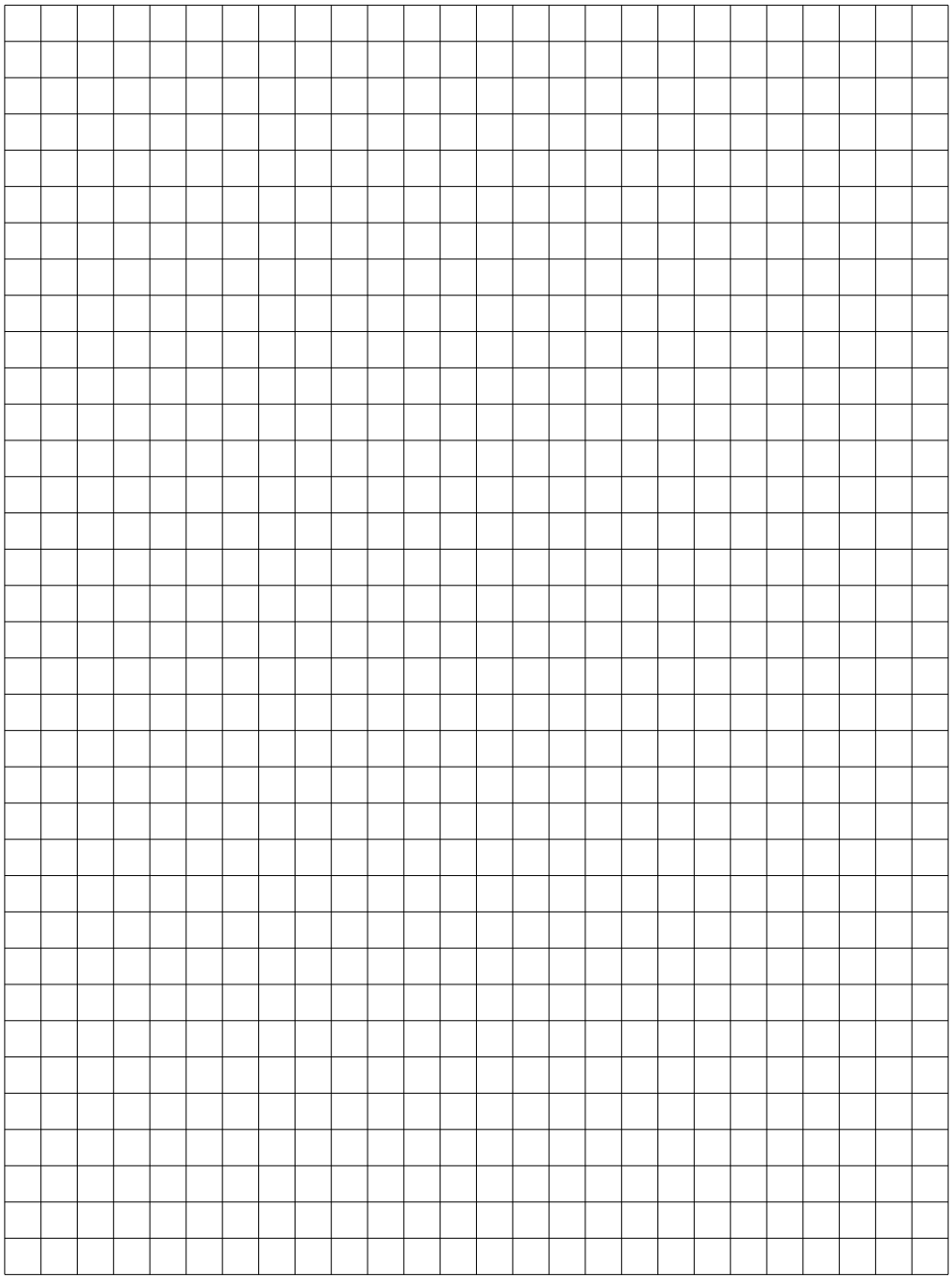
The Orienteering Problem (OP) is a graph problem in which the objective is to find a path between two given nodes, maximizing the total score collected from visited nodes given a limited time budget. Applications of the OP are varying from sales management to logistics of fleet of vehicles. In this lecture I will show a genetic algorithm finding near-optimal solutions to OP using real-life distances of tourist attractions in the city of Białystok.

MSC (2020): 68W50, 90B06





Fractional Calculus Unified Across Time Scales for Dynamical Systems

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Fractional calculus on time scales has advanced considerably over the past two decades, yet existing formulations remain largely operator or kernel-driven and do not fully preserve the intrinsic structure of arbitrary time scales. In particular, the interaction between graininess, jump operators, and the domain over which "fractional memory" is accumulated is not treated consistently, leading to implicit domain shifts in discrete and non-uniform settings and preventing a genuine unification of continuous and discrete fractional theories.

In this talk, we introduce a new structural framework for fractional calculus on time scales based on a graininess-dependent fractional operator that explicitly maps between time scale domains. This provides the geometric foundation for a cross-domain convolution theory, from which fractional kernels, integrals, derivatives, and transform methods are developed in a unified and consistent manner. The resulting framework recovers the classical continuous and discrete fractional operators as natural special cases while extending seamlessly to arbitrary, hybrid, and highly non-uniform time scales.

Building on this foundation, we establish corresponding theory of Laplace transform representations together with existence and uniqueness results for fractional dynamic equations. The framework also provides a natural pathway for extending fractional calculus to fractal time scales, including the Cantor set, and for formulating fractional partial dynamic equations on general time scales. These developments offer a unified mathematical foundation for modelling complex systems whose dynamics combine continuous, discrete, hybrid, and fractal behaviour.

MSC (2020): 26A33, 34A08, 34K37, 34N05, 44A55

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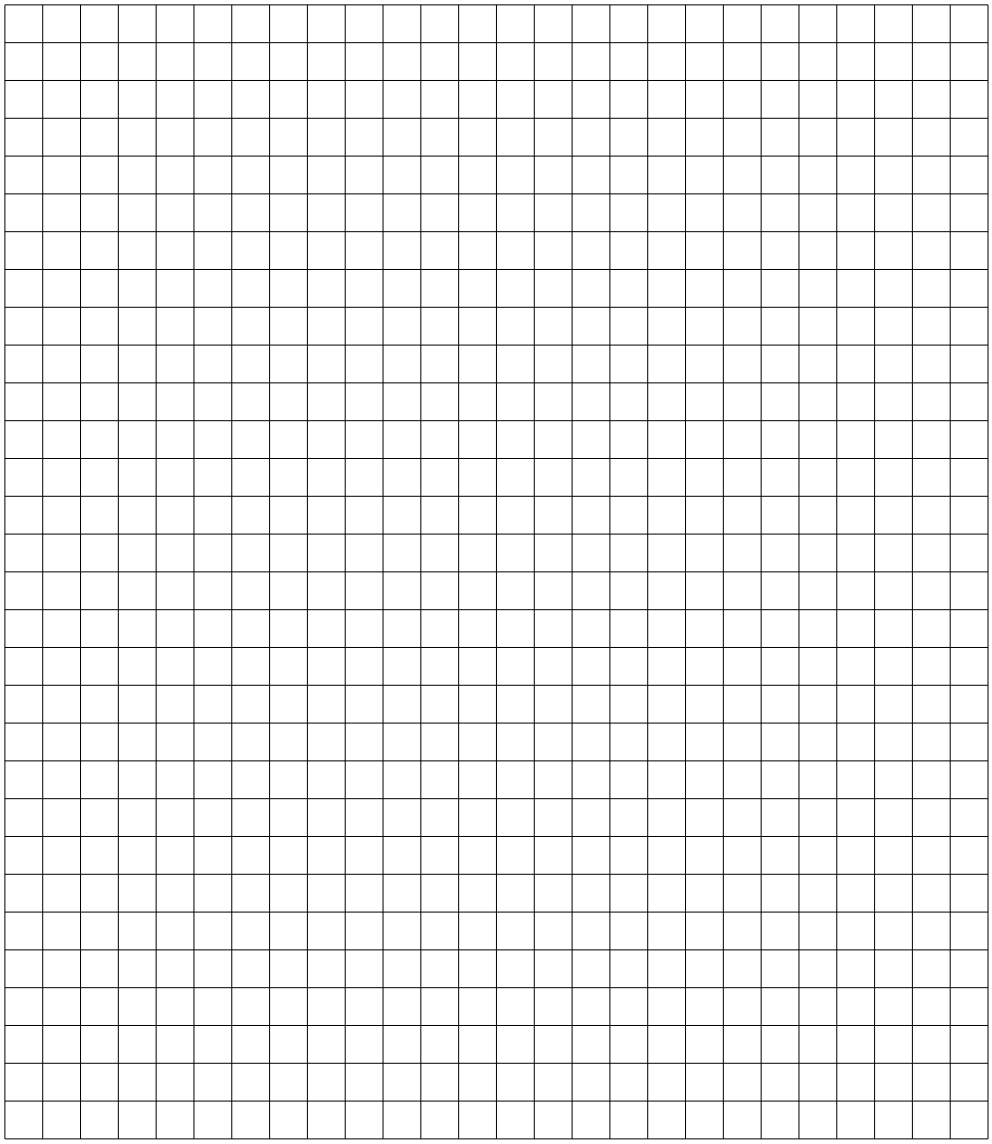

A Fuzzy Belief-Based TOPSIS Framework for Ordinal Survey Data Analysis: An Application to Airline Customer Satisfaction

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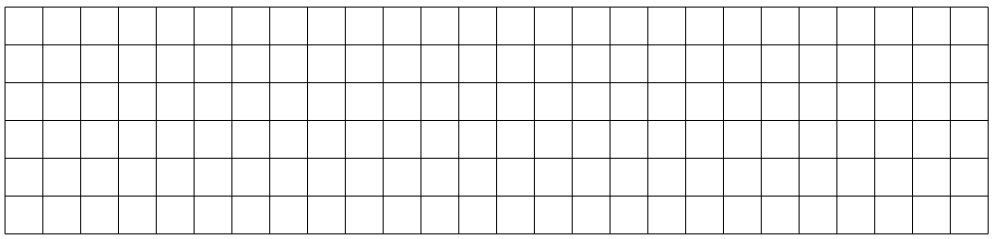
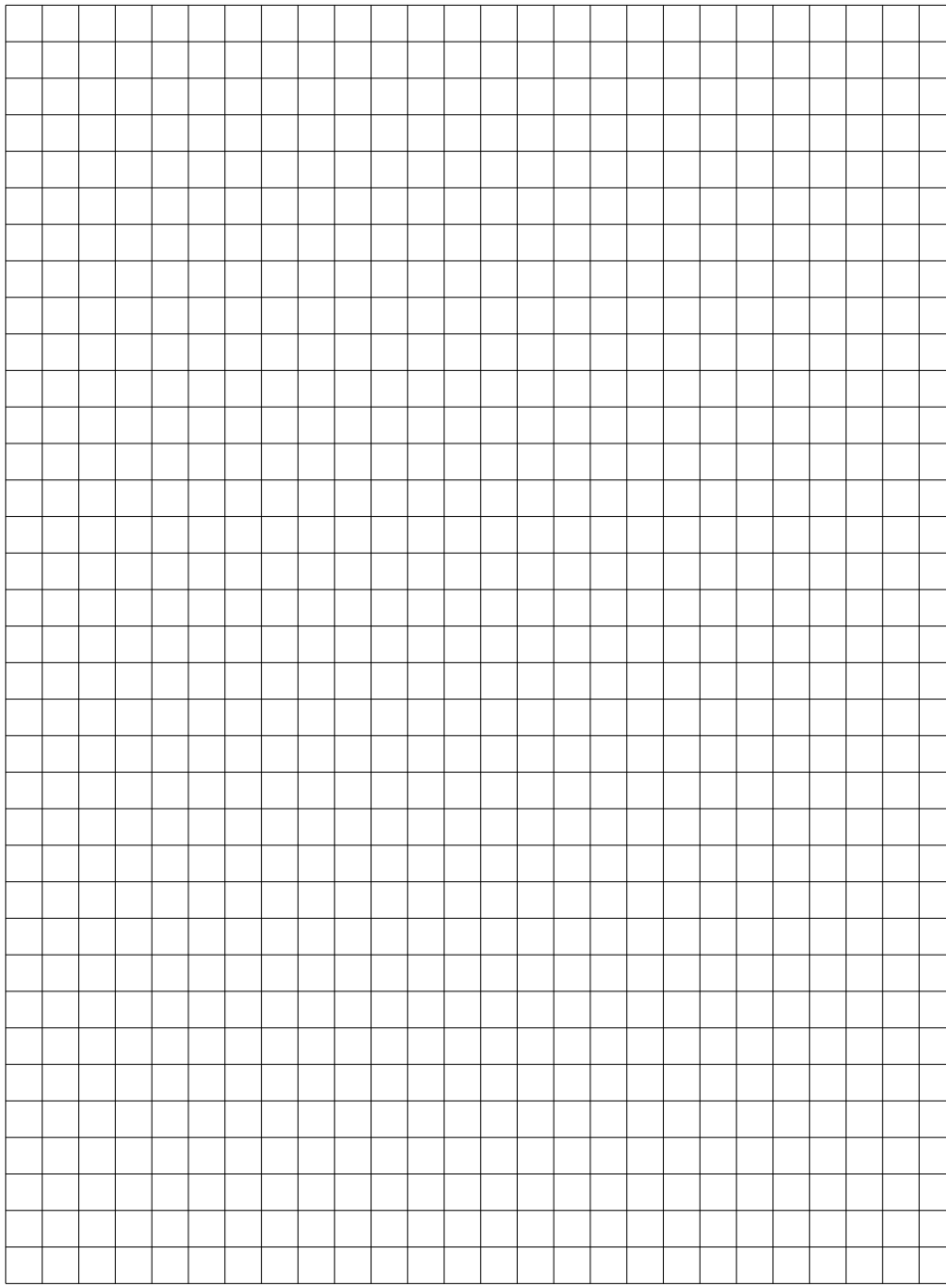
Ordinal survey data expressed through linguistic categories are inherently imprecise and require aggregation methods that preserve their semantic meaning. This paper extends the Fuzzy Belief Structure–based TOPSIS (FB-TOPSIS) approach by combining fuzzy utility functions with belief structures for analysing ordinal evaluations. Respondents' opinions are represented as fuzzy belief distributions over predefined satisfaction levels using triangular and trapezoidal fuzzy utilities. The method is applied to TripAdvisor reviews of 25 Star Alliance airlines to derive an Airline Satisfaction Index (ASI). The results identify Singapore Airlines, All Nippon Airways, EVA Air, and Air New Zealand as the highest-ranked airlines. The study demonstrates that FB-TOPSIS provides an effective tool for analysing linguistic survey data and supports decision-making in multi-criteria evaluation problems.

MSC (2020): 90B50, 03E72

*Presenting author.





Packing Problems in Additive Manufacturing

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Additive Manufacturing is an extremely complex process. Hence, optimization models and methods play a central role for improving related technologies to save resources like material, energy, and time, or to satisfy further objectives. Especially, a bunch of packing problems arises, which is exemplified by some use cases.

The first is the generation of void structures in 3D parts for topology optimization. This allows to reduce the weight of these parts as well as printing time and can be modeled as a packing problem depending on the number, shapes, and positions of the holes under constraints that also guarantee sufficient mechanical strength.

Another case is the placement of as many parts as possible in the printer chamber to increase efficiency. This leads to a packing problem where an allowable distance between the parts has to be ensured to avoid that parts stick together or deformations occur during cooling after the printing process.

The third case concerns mixing various-sized spherical powders of titanium alloys. To achieve desired mechanical properties, a ratio between different powder fractions has to be maintained. An algorithmic layer-by-layer approach can be successfully employed to deal with the underlying problem of packing the various-sized particles.
All packing problems presented above are nonconvex optimization problems. Practical instances are large in terms of the number of parts and constraints. The development of solution methods is a challenge in respect to solution time. Therefore, modeling aspects, algorithmic approaches, and numerical experiments become extremely important for solving those packing problems numerically. In particular, this includes the analytical description of placement conditions by means of phi-function techniques and a combination of model-based heuristic approaches with decomposition methods.
For a detailed and comprehensive presentation, see [1].

MSC (2020): 90C90, 90C06, 90C59, 49M27

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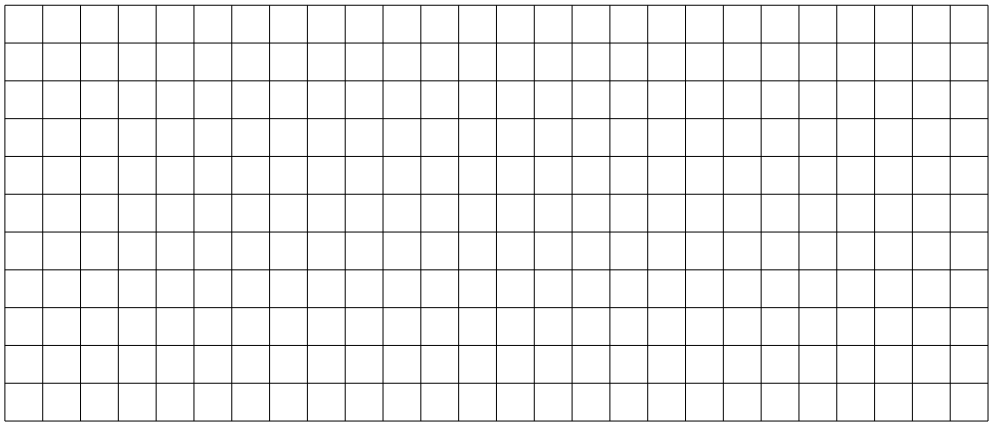
*Presenting author.



Multi-Objective Optimization of Reward Function Weights for Reinforcement Learning Control of a Snake-Like Robot

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Snake-like robots as an area of robotics have been studied since the 1970s, with early work focusing on mechanical design, biologically inspired locomotion, and hand-made control strategies [1]. Over time, many approaches to controlling highly articulated snake-like mechanisms were proposed, ranging from analytically designed gaits to model-based and model-free control methods [2]. Recently, improvements in simulation environments and reinforcement learning have allowed neural-network-based controllers to be trained directly in physics simulation, making them especially attractive for robots with many degrees of freedom [3].

This presentation discusses an attempt to optimize the reward-function weights used during reinforcement learning of a 32-degree-of-freedom snake-like robot controller. The robot was trained in NVIDIA Isaac Lab using PPO, while an external NSGA-II optimization loop searched for reward weights that would encourage forward motion in the negative X direction while limiting lateral drift. The optimization treated each reward-weight vector as a candidate solution: for every candidate, a policy was trained, evaluated deterministically, and scored using two objectives related to final forward displacement and side deviation.

The work showed that the reward design strongly affects learned locomotion. An initial optimization based mainly on center-of-geometry motion led to reward exploitation, where the robot achieved favorable numerical scores by curling or deforming its body rather than producing useful locomotion. A second optimization modified the reward to emphasize head velocity while still penalizing lateral motion and unstable behavior. This produced a Pareto front of trade-offs and identified a candidate, which generated a slow but repeatable crawling pattern with a visible wave of motion along the body.

Although the resulting controller was not yet fast, it demonstrated that multi-objective optimization can help identify reward parameters that lead to more structured locomotion than manual tuning alone.

MSC (2020): 68T40

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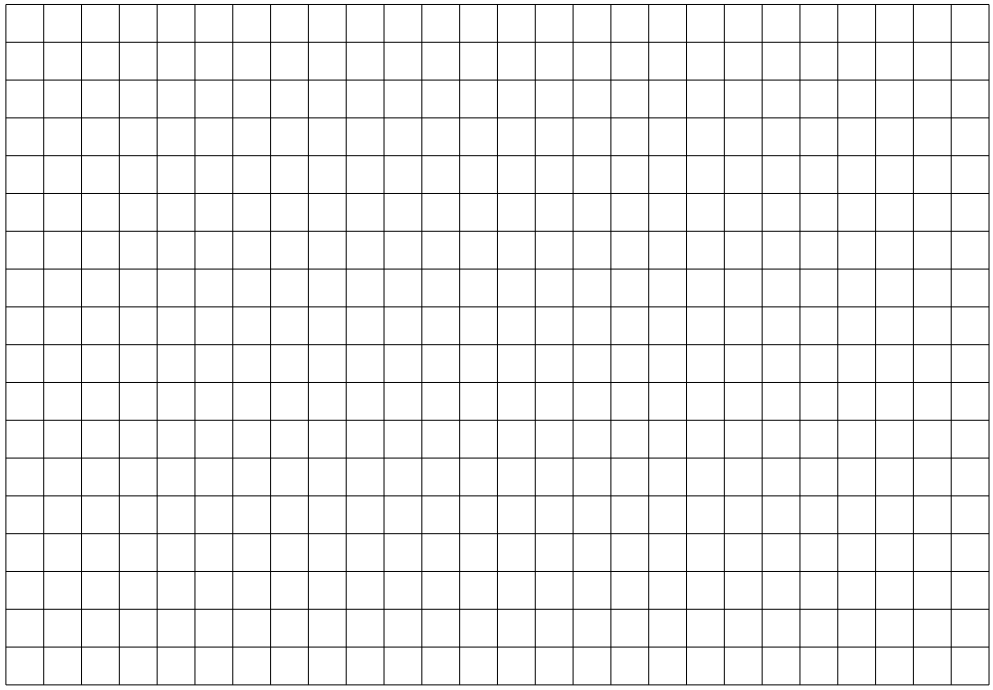

Finite-Time Stability Analysis of Switched Systems Involving Caputo Fractional Derivatives with Respect to Arbitrary Kernels

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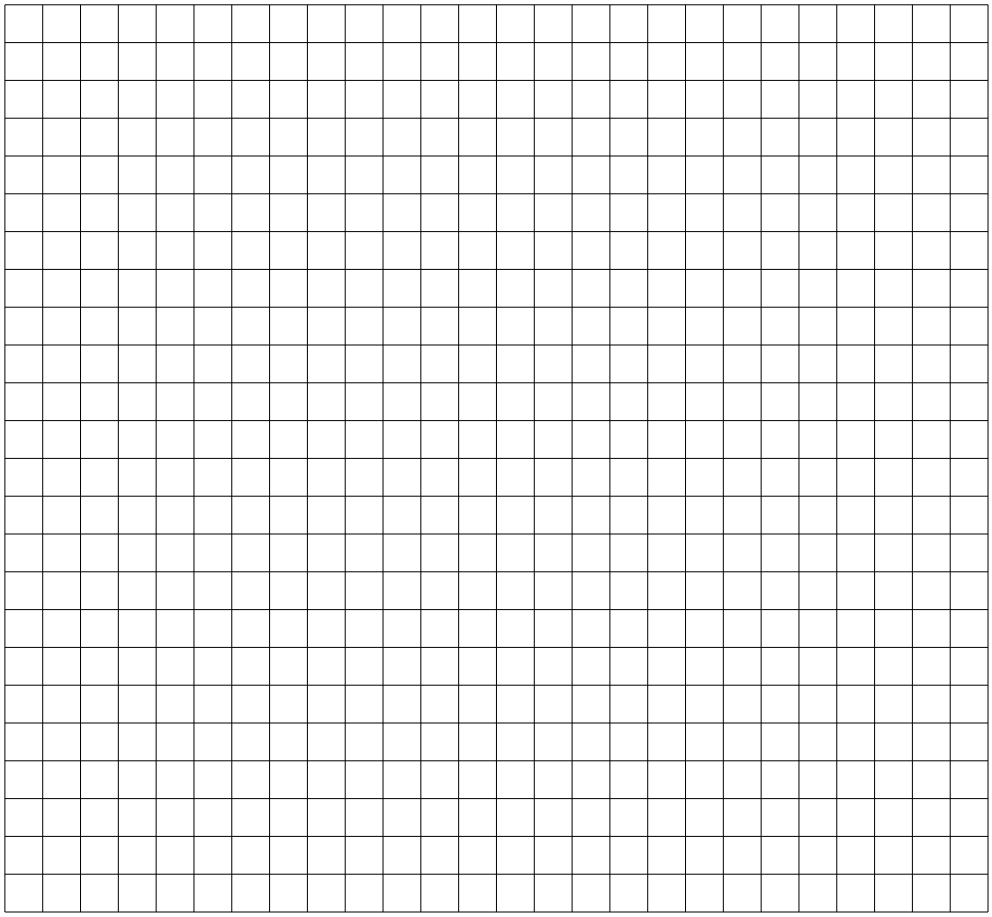
During the talk the finite-time stability (FTS) of a class of switched fractional-order systems described by the Caputo fractional derivative with respect to another function will be investigated. The considered model involves a family of linear subsystems with external inputs and a piecewise constant switching signal over a finite time interval. Sufficient conditions ensuring finite-time stability with respect to prescribed bounds and a finite horizon are derived for systems subject to bounded inputs. In particular, explicit inequalities involving system parameters, input bounds, and the kernel-dependent time measure

are established to guarantee that the state trajectory remains within a desired bound over the interval of interest. Two alternative FTS conditions are provided, leading to tractable tests based on computable constants. The theoretical results are illustrated by numerical examples, including an electrical system model, demonstrating the effectiveness of the proposed approach.

MSC (2020): 26A33, 34A08, 93D09, 93D20

*Presenting author.



Optimizing Speech Codec for Automatic Speech Recognition on Resource-Constrained IoT Nodes

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The Internet of Things (IoT) systems are gaining popularity in a variety of fields, including industry, medicine, traffic control, and perimeter surveillance. These systems can cover large areas, with distances between nodes sometimes reaching several kilometers. Remote nodes often encounter challenges related to their power supply, as they typically depend on renewable energy sources like solar panels, which can limit their power availability. As a result, many of these applications make use of resource-constrained microcontrollers.
Wireless communication in IoT systems that cover large areas encounters significant challenges, especially in maintaining very low power consumption. These protocols often provide low data bandwidth, which diminishes even further with increased distance. This issue is particularly critical for applications that require speech transmission. One potential solution is to implement efficient speech compression that demands minimal resources.
Contemporary Internet of Things (IoT) systems commonly utilize artificial intelligence models for various tasks, including data preprocessing, classification, image recognition, and automatic speech recognition (ASR). In applications involving ASR, it is crucial to transmit data with minimal noise and distortion. However, the most effective speech codecs tend to be lossy, which can introduce significant distortion. This research addresses the problem of optimal selection of speech codec factors to improve the quality of automatic speech recognition. Current research focuses on key aspects of how speech compression impacts automatic speech recognition quality. Some current work include:

- the work [3] analyzed the effect of the Opus codec at various bitrates on multi-channel ASR;
- the works [4] and [5], where the authors conducted experiments on low-bitrate, packet loss, and ASR resilience;
- the work [6] presenting an interesting approach to enhancing codec performance by using deep learning techniques.

The main drawback of mentioned solutions is very big demand on resources. The proposed speech compression algorithm is a lossy codec that employs differential sample encoding with error distribution, building on the previous research conducted by the author [1, 2]. The coding process involves several straightforward data processing steps. Experiments show that after coding and decoding, the resulting audio stream can be effectively utilized by a speech recognition system. Furthermore, the processor utilization is currently at a very low level, which results in minimal power dissipation. Research identifies potential areas for optimization and algorithms for selecting factors used in either the entire data stream or in blocks of specified sizes from a diverse range.

MSC (2020): 65G30, 65K10, 65Y20, 68P30, 68T10

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Mean Field Games with Common Noise Applied to Smart Grids

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In a smart grid, consumers may also play the role of producers and sellers of energy. Therefore, the control of a smart grid over time becomes a dynamic game with a large number of small players and one or several large players. This is much more complicated than the control of an energy system with a single producer. The framework of meanfield games with common noise [3] allows the transformation of such a complicated model into a control problem for a typical prosumer (producer and consumer in one) optimizing his behavior against a flow of other such prosumers. In work [2] authors had similar approach, that focus on optimal charging of electric vehicle batteries, but set prices in different form, while authors of [1] focused on trading energy in market and satisfying terminal delivery of electricity demand. In our model we connect these ideas and introduce 2 decision variables for such a controller: one describing the amount of electric energy bought from the grid (or sold to it), with the second one describing the charging rate for our battery. The cost function is designed in order to satisfy two goals: satisfying the demand of the consumer (a single house) and minimizing the payment for energy. Players are allowed to sell/buy energy from the grid. Common noise is hidden in price volatility and sun irradiance, which influence all the players. We compute the optimal controls for our model and simulate solutions on real data. The novelty of our construction when compared to other works on the control of smart grids lies in combining the minimization of cost of energy and sustaining some demand satisfaction level. It turns out that using batteries is crucial in obtaining a balance between these two criteria.

MSC (2020): 49N80

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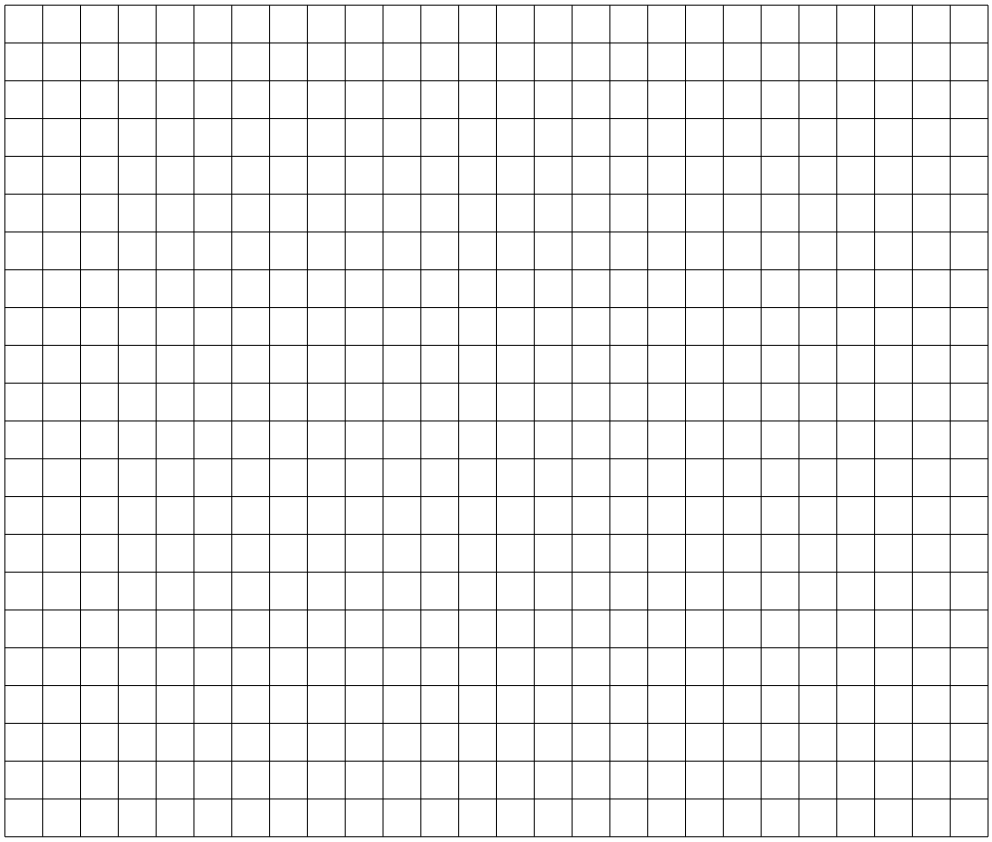

The Curious Case of Surface Optimization and Soap Bubbles

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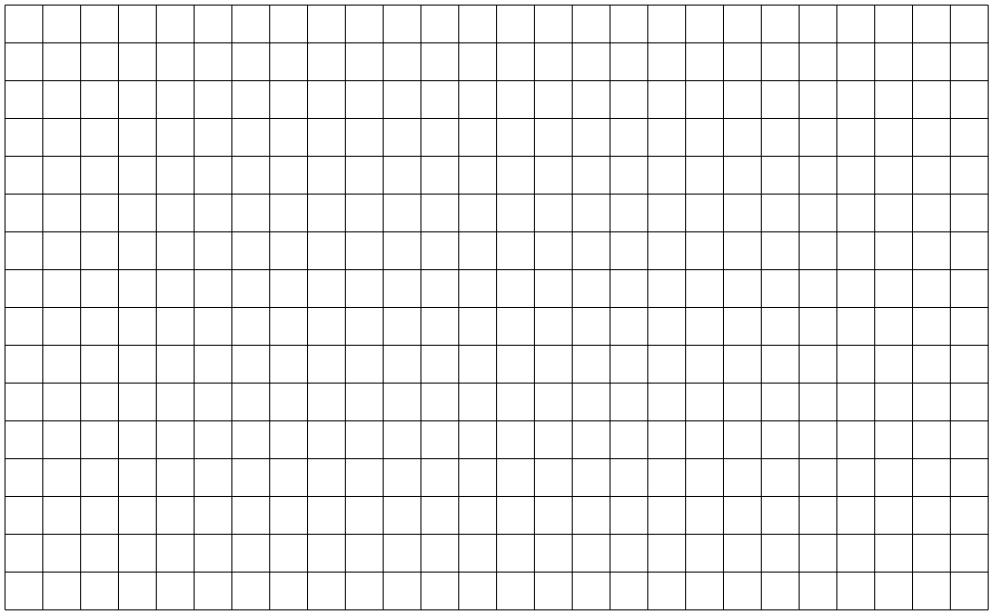
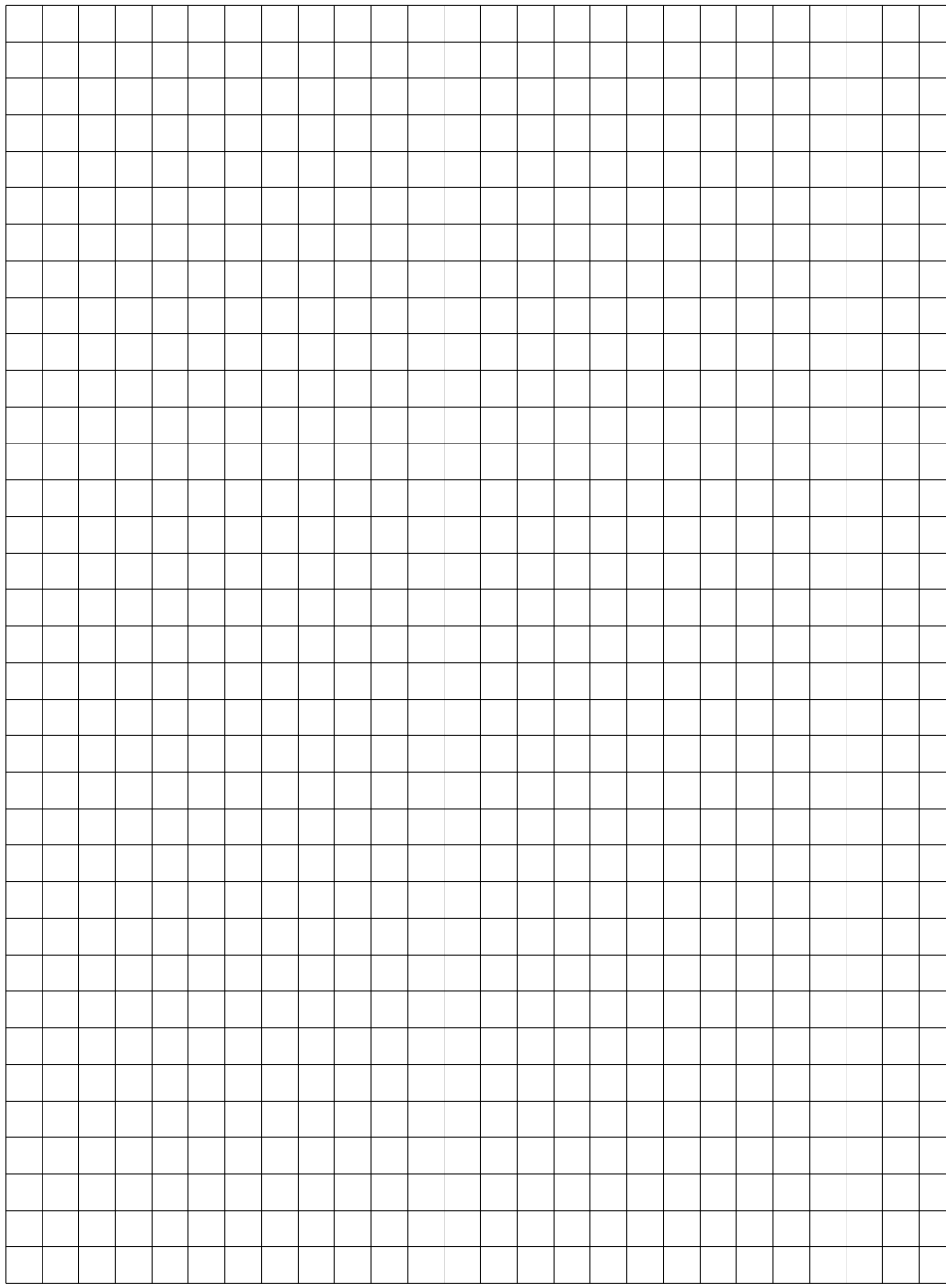
The optimization of geometric structures plays a central role in both mathematics and modern engineering. A particularly elegant illustration of such problems is provided by soap bubbles, which naturally minimize surface area under given constraints. This presentation explores stated phenomena through the lens of the Calculus of Variations, Euler-Lagrange equation and Jacobi's sufficiency condition, focusing on the determination of optimal surfaces constrained by wireframes. Particular attention will be given to the existence and character of extrema, the question of whether a minimum is strict, and the stability of the resulting configurations.

MSC (2020): 49Q05





Covering Results for Constrained Equations Beyond Metric Regularity

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Covering properties build the foundation of stability and sensitivity analysis of solutions to a constrained equation and more specific stationarity and equilibrium problems. It has been well-understood that the concept of metric regularity is key to furnishing Lipschitzian stability of the solution of interest [1]. In this talk, we want to leave the ties of metric regularity and, based on [2], formulate novel covering results for constrained equations under the concept of metric 2-regularity [3]. Among other things, our findings target a precise description of the set with nonempty interior being covered. We will also apply our observations to coupled constraint systems arising in practically relevant settings, including variational and optimality systems [3].

MSC (2020): 47H04, 49J53, 49K40, 90C31

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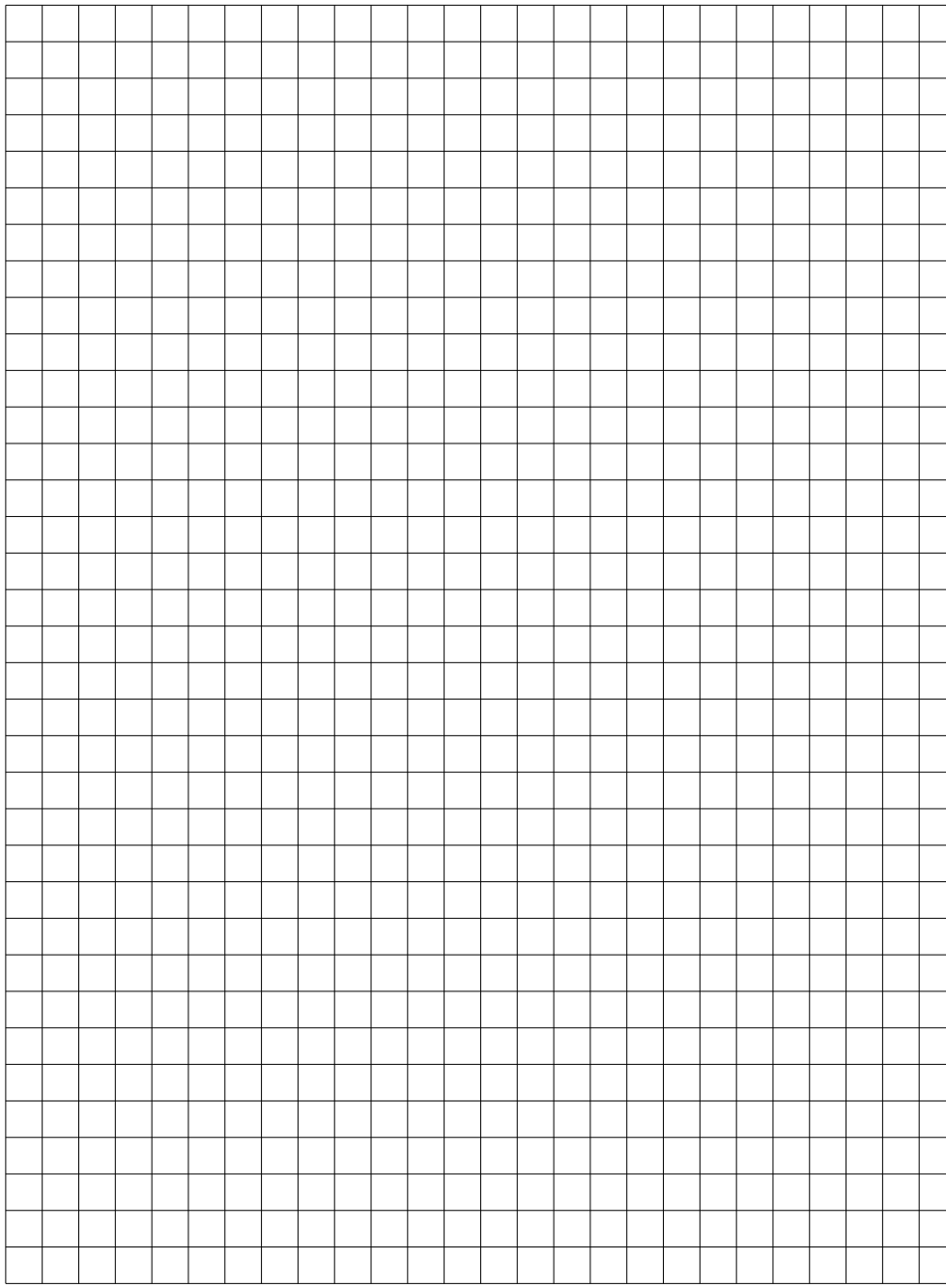

Parameter Optimization of the MCTS Algorithm for the Combinatorial n^d game

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Monte Carlo Tree Search (MCTS) [2, 5] is a decision-tree search algorithm based on the Monte Carlo method, widely utilized in machine learning and for determining optimal strategies in combinatorial games. Due to the high complexity of the multi-dimensional variant of tic-tac-toe, known as the n^d game [3, 4], classical deterministic methods such as min-max or alpha-beta pruning prove inefficient, necessitating the use of approximation approaches. This paper addresses the problem of selecting optimal parameters for the MCTS algorithm [1], specifically the number of iterations and the exploration parameter, to maximize the agent's decision-making efficiency and align its strategy as closely as possible with optimal solutions.

MSC (2020): 68T20, 68W20, 91A46

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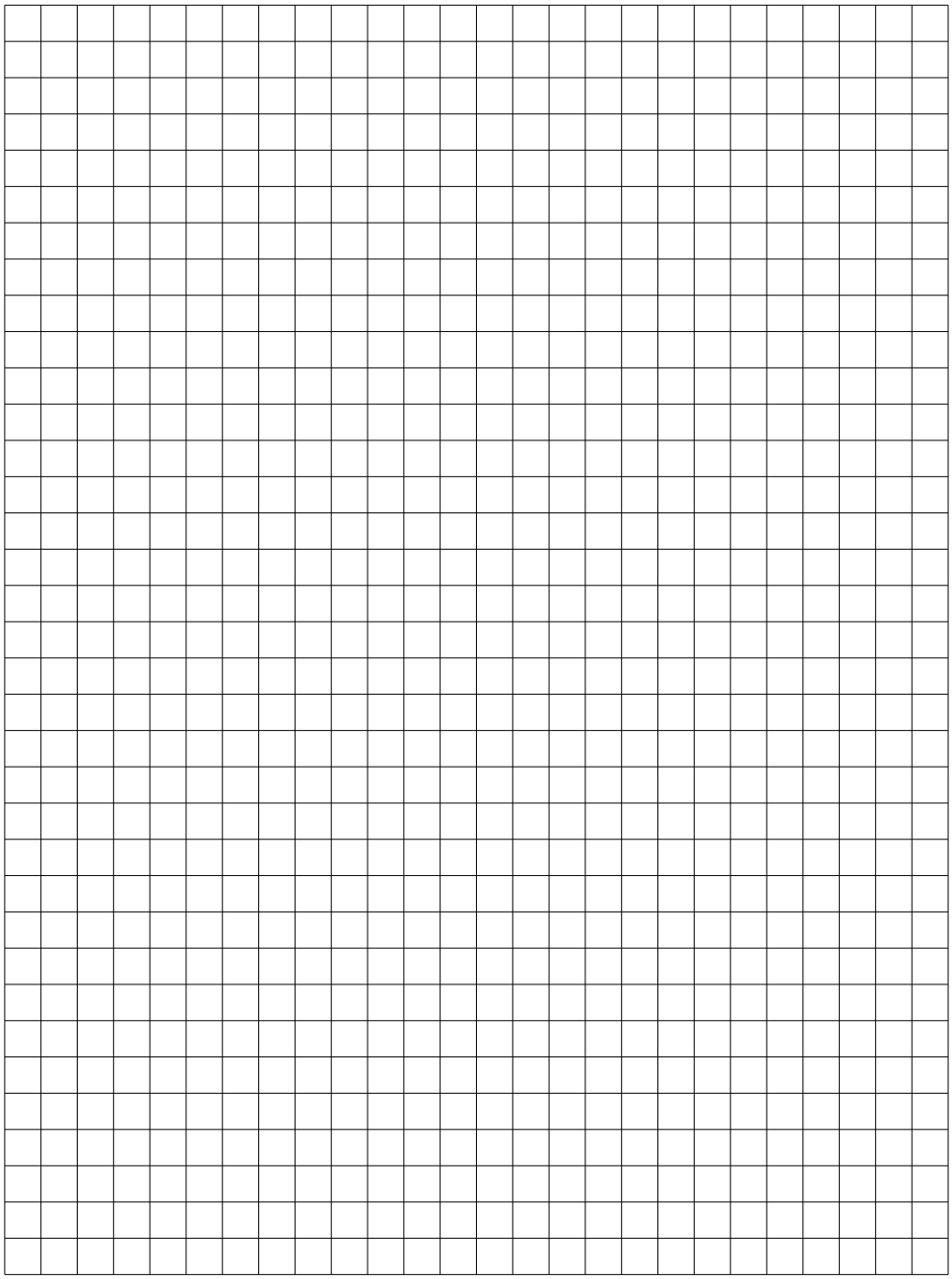

Multi-Objective Optimization of Hyperparameters and Feature Set Size for File Encryption Algorithm Identification

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Identifying the encryption algorithm used to protect a file is relevant to digital forensics and automated security analysis. Among the various possible solutions, machine learning plays a notable role in distinguishing selected cryptographic algorithms from statistical properties of ciphertext [2]. The previous author's publication [1] examined six file formats and both unencrypted files and files encrypted with AES, DES, 3DES, and RC4. For the block ciphers, Electronic Codebook (ECB) and Cipher Block Chaining (CBC) modes were considered, together with different numbers of keys and dataset sizes. Files were described by basic statistics and byte frequency distributions, without using headers, keys, or internal structure. Among six classifiers, Random Forest and Bagging achieved the highest performance and stability. Results differed between ECB and CBC, but confirmed identification from the adopted representation in both modes.

The planned study extends this analysis by selecting hyperparameters and feature count simultaneously [3]. Its objectives are to maximize macro-F1 and limit the features required for classification. Random Forest, Bagging, and Support Vector Machine will be examined. The first two follow from the previous results, whereas SVM defines class boundaries differently.

Features will be ranked using mutual information, and the models will then be evaluated on subsets of different sizes containing the highest-ranked features [6]. The ranking will be calculated separately in each training fold of cross-validation so that validation data do not influence feature selection or model assessment [7].

NSGA-II [4] will be used to search for trade-offs between classification performance and feature count. Its results will be compared with Random Search [5] using the same number of evaluated configurations. Experiments for ECB and CBC will be conducted separately, and default classifier configurations will

provide an additional baseline. The evaluation will include macro-F1, feature count, training and prediction time, model size, and feature-ranking stability. The expected outcome is a set of configurations describing trade-offs between performance and feature count. These results will show whether the models can be simplified without substantially reducing classification performance and whether the required feature count differs between ECB and CBC. Any differences will be determined experimentally. The results may support configuration selection under computational constraints.

MSC (2020): 68T05, 68W50

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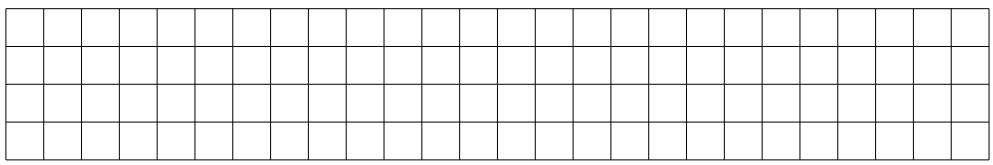

Stabilization of the Ramsey Growth Model with Controlled Population Dynamics

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We study the stabilization of the Ramsey growth model in which demographic dynamics can be influenced through a population policy intervention. Capital and consumption *per capita* follow the standard Ramsey dynamics, while population is governed by a logistic growth law and an additional external control input. This policy intervention may be interpreted, for example, as immigration policies or fertility incentives.
Further, we propose a data-driven method for constructing near-optimal feedback laws for time-optimal control problems. We compute the switching surface that determines the stabilizing bang-bang feedback law.
The results show that, for the Ramsey model, a single demographic control is sufficient to stabilize the coupled economic and population dynamics near the equilibrium.

MSC (2020): 00A69, 49M05, 49M25

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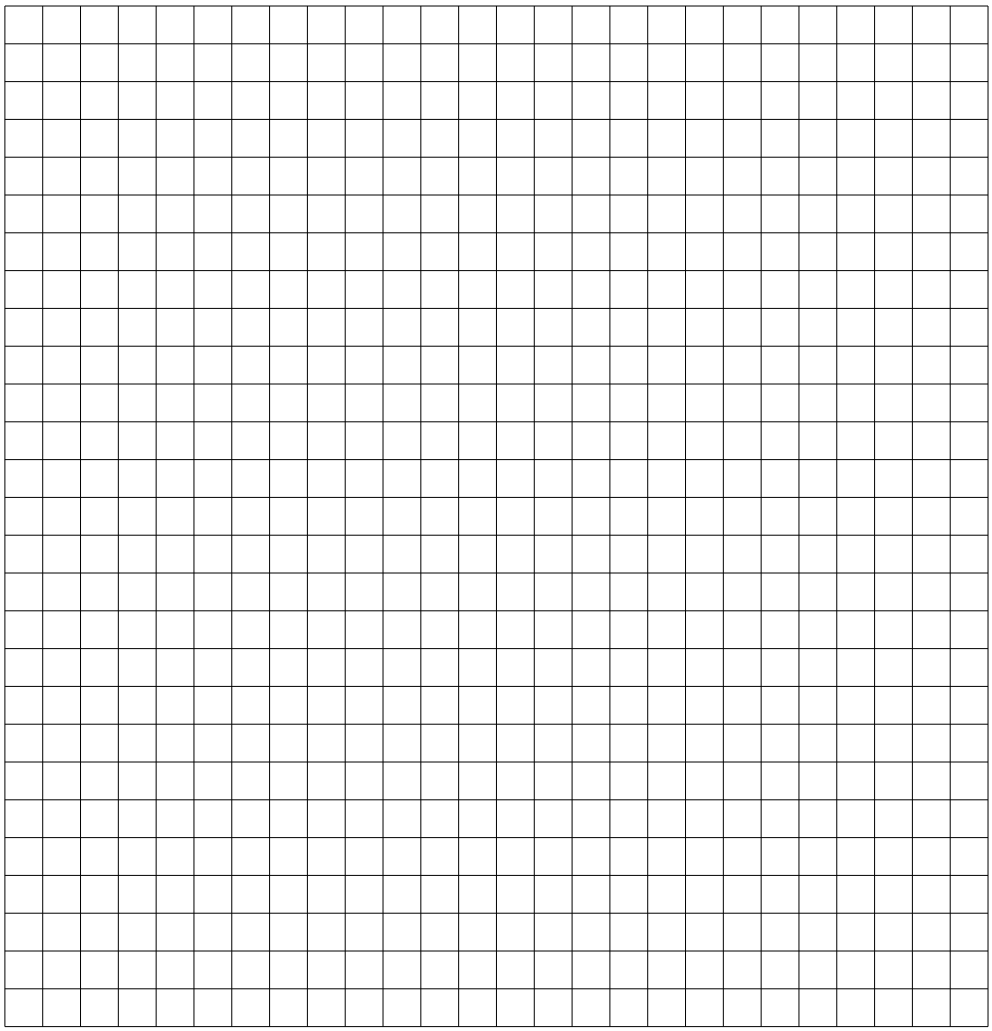

Second-Order Conditions Beyond Surjectivity

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The talk concerns second-order necessary optimality conditions for smooth equality-constrained optimization problems in Banach spaces when the derivative of the constraint mapping is not assumed to be onto. This situation appears naturally for differential constraints.
The talk starts from the generalized Mangasarian-Fromovitz viewpoint for degenerate equality constraints, where rank-theorem geometry replaces the usual surjectivity requirement. In the pure equality case, a C^2 constant-rank hypothesis locally straightens the feasible set. This makes it possible to realize algebraic second-order corrections as genuine feasible second-order arcs. Consequently one obtains the primal second-order necessary condition, and, using the closed-range consequence of the rank theorem, also its Lagrange multiplier form.
The aim is to present the main geometric mechanism and to indicate how differential constraints with compatibility conditions may be used to illustrate it. In such problems Robinson regularity can fail in the chosen constraint space, while a constant-rank structure may still remain available.

MSC (2020): 49J53, 49K27, 46N10, 58C15

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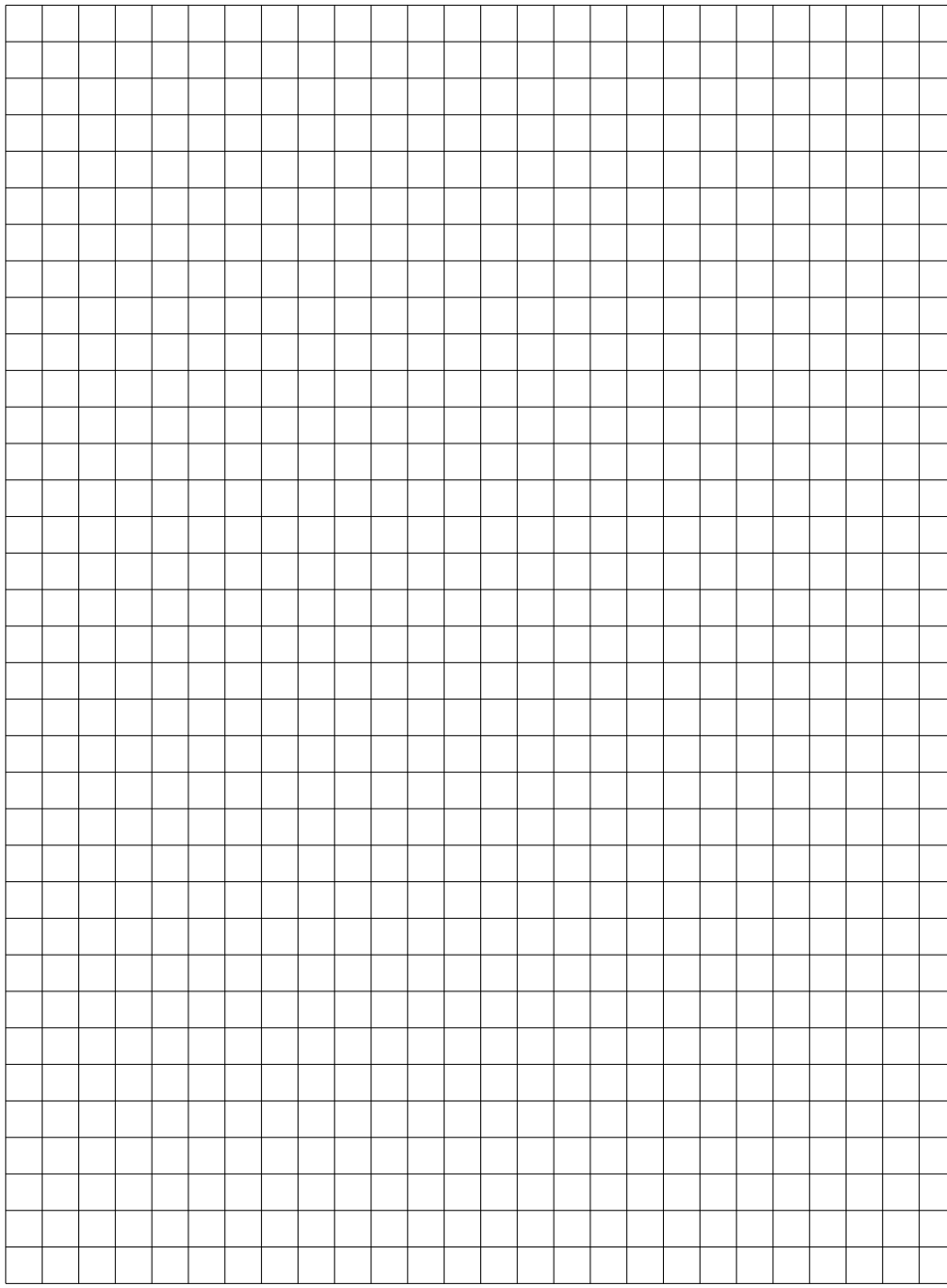

Computing all Nash Equilibria of Low-Rank Bi-matrix Games

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Low-rank bi-matrix games are transformed into typically smaller bi-matrix games, where the constraints for both players are arbitrary polytopes rather than simplices. These constrained bi-matrix games, also called polytope games, are studied, in particular, all Nash equilibria are computed using bensolve, a solver for multi-objective linear programs. The connection between the Nash equilibria of the original game and the smaller polytope game is investigated.

MSC (2020): 91A05

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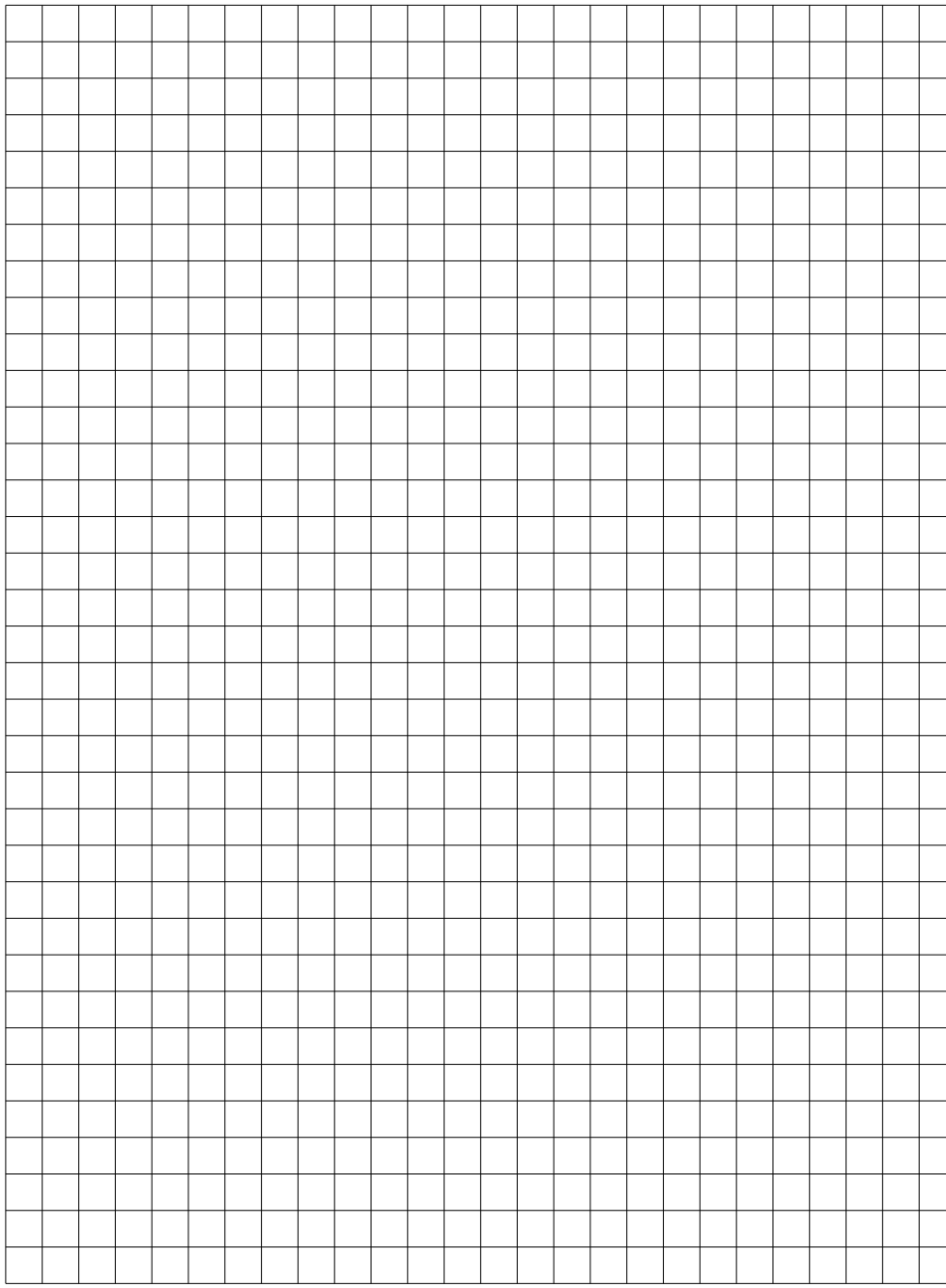

Genetic Algorithms as a Combinatorial Optimization Method for Solving Sudoku

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Solving Sudoku puzzles is a classic example of an NP-complete problem in combinatorial optimization [1]. While traditional deterministic methods, such as backtracking, work well for simple instances, their computational complexity scales poorly as difficulty increases. This presentation introduces the core concept of using genetic algorithms as a heuristic approach to tackle this problem [2]. It explains how evolutionary mechanisms can be adapted to the geometry of Sudoku, especially by focusing on the fitness function used to evaluate the board [3]. The talk covers fundamental theoretical aspects, highlighting the potential of evolutionary methods in large state spaces, as well as practical challenges like the risk of premature convergence [4].

MSC (2020): 68T20, 68W50, 90C27

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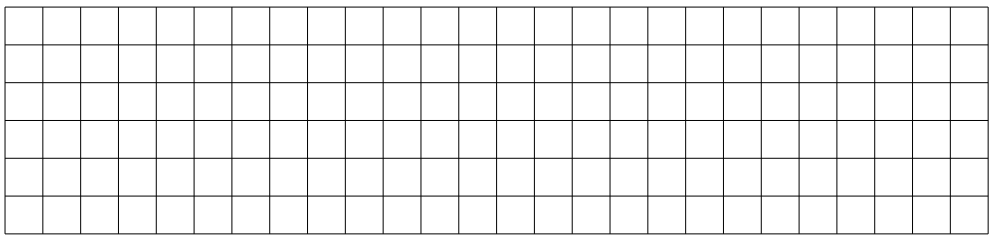

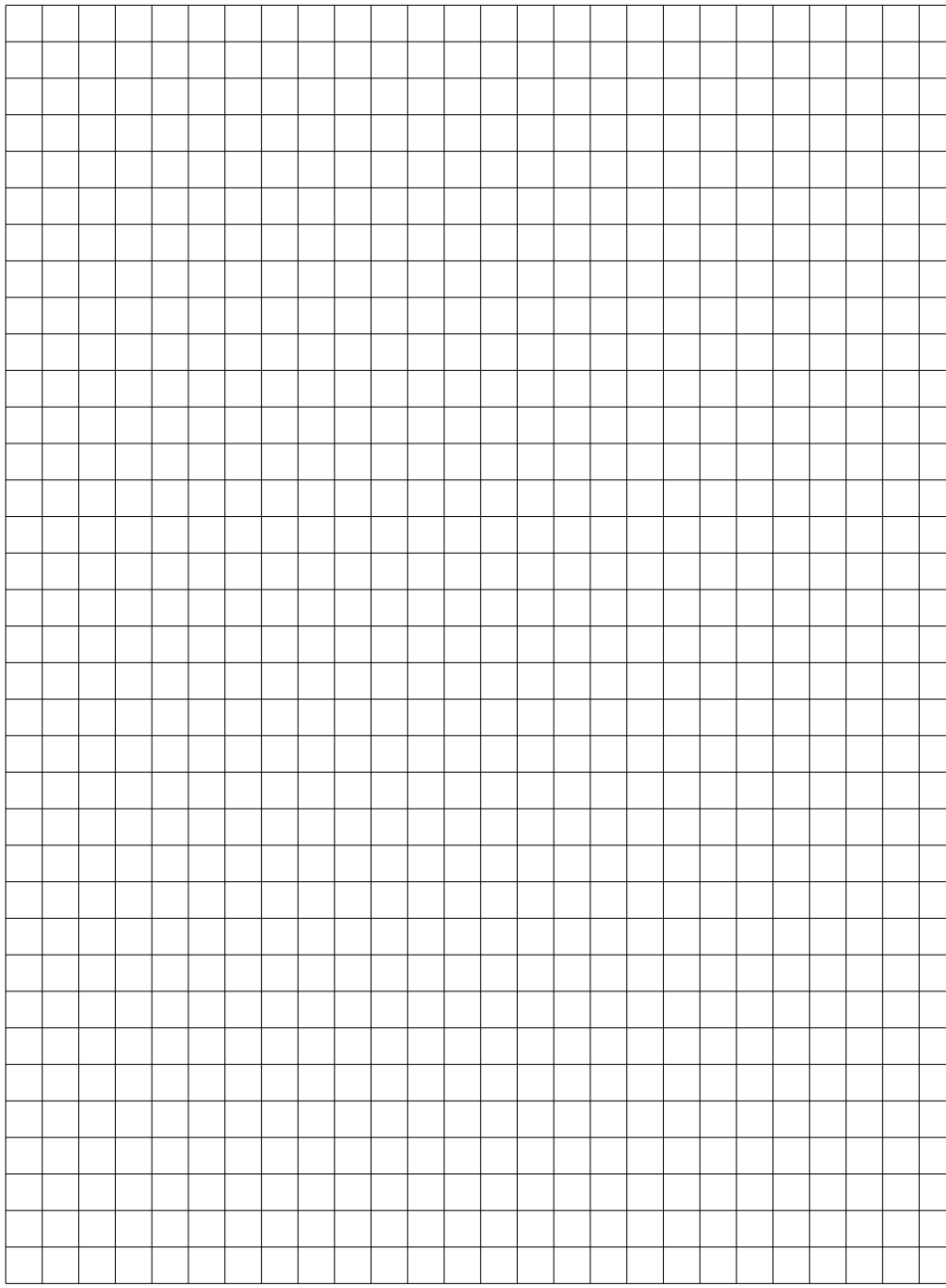

Generalized Choquet Integrals for Enhancing Classical Machine Learning via Non-Additive Feature Modeling in Eye-Tracking-Based UX Behavioral Classification

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This paper presents a generalized aggregation framework for behavioral classification using eye-tracking data, combining interaction-aware modeling with locally smoothed input representations. By incorporating concepts inspired by numerical quadrature, the approach accounts for feature dependencies and improves robustness to noise and user variability [1]. The proposed method is evaluated in a user experience (UX) [2] classification setting. Although

standard models yield similar baseline results, combining them through non-additive aggregation yields noticeable performance gains. The most effective variant relies on a smoothed formulation with a tunable interaction mechanism, achieving both high accuracy and consistent results across repeated experiments [3]. The study highlights the importance of jointly considering feature interactions and stability-oriented transformations when aggregating model outputs. This approach incurs additional computational cost and relies on data-driven parameter tuning. It offers a flexible and effective alternative to traditional aggregation schemes in eye-tracking-based analysis. Additionally, such aggregation can significantly reduce classification uncertainty by improving task quality.

MSC (2020): 28E10, 62H30

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On the Stability of Memory-Dependent Systems

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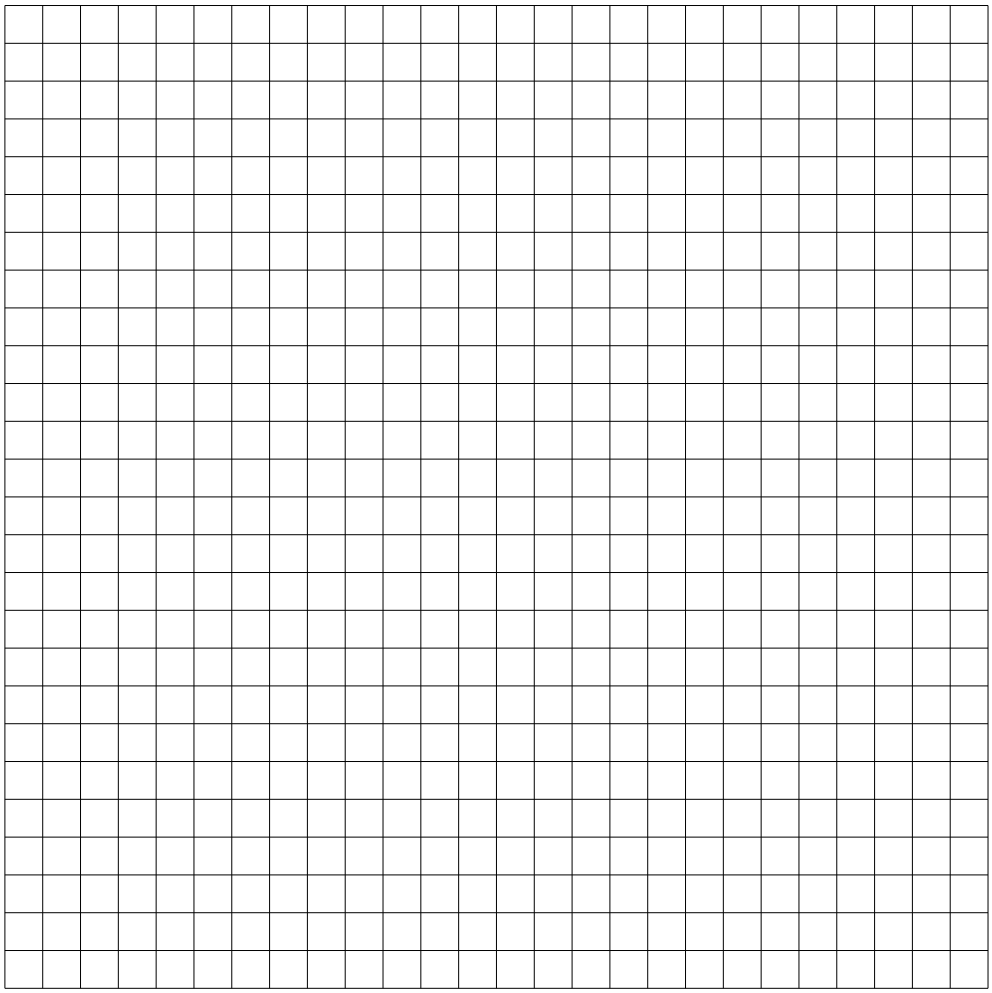
This talk is organized into two parts.
The first part addresses the positivity and stability of fractional differential systems. We present necessary and sufficient conditions for the positivity of such systems, together with a sufficient condition for the asymptotic stability of positive linear fractional switched systems under arbitrary switching. These theoretical results are subsequently applied to analyze the asymptotic stability of multi-agent systems operating under Denial-of-Service (DoS) attacks.
The second part focuses on the global exponential stability of switched systems described by linear fractional difference equations. A new framework based on practical stability with a fixed memory length L is introduced. Sufficient stability conditions are derived using the concepts of average dwell time and

chatter bound. Finally, numerical examples are presented to illustrate the effectiveness of the proposed approach and to validate the theoretical results.

MSC (2020): 26A33, 34A08, 39A30

*Presenting author.



Optimizing Traffic Flow in Peri-urban Road Networks through Multi-agent SUMO Simulation: The Effect of Intelligent Vehicle Penetration with Dynamic Rerouting

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Peri-urban road networks are roads that, unlike strictly urban environments, combine high speeds and large capacity with a small but critical set of junctions where bottlenecks concentrate. Recurrent congestion in these areas has a negative impact on metropolitan mobility, resulting in increased travel times, higher fuel consumption, and greater pollutant emissions. Thus, smart mobility is required to reroute traffic when congestion appears, in order to guarantee mobility and road safety. The main purpose of this work is to determine the relevance of smart traffic rerouting to optimize travel times when congestion appears in this kind of road network.

This work presents a multi-agent traffic simulation framework, built on the SUMO traffic simulator and driven through its TraCI interface [3], to optimize flow in such networks and to assess how different penetration rates of intelligent vehicles affect it. The framework builds on previous work on dynamic itinerary optimization and SUMO-based traffic assessment [1], [2].

Each vehicle is modeled as an intelligent agent. Its knowledge includes the network traffic status, via Vehicle-to-Everything (V2X) communications, and a set of routing algorithms. The intelligent algorithm is based on the traditional A* algorithm [4] and combines different traffic information to determine the best route, attending to the real-time traffic situation from its current location to its destination. This route calculation is performed at every junction.

A real test-bed has been implemented to analyze the impact of the number of vehicles using the intelligent algorithm. The test-bed comprises a road network of 80 km in length with four main routes (one motorway, two national roads, and one regional road) and eight alternative routes. The road demand is based on a regular traffic flow. A set of six scenarios has been simulated, varying both the road network configuration and the occurrence of an incident, so as to evaluate the framework under different operating conditions.

Furthermore, a severe incident is simulated on the motorway, blocking one of its lanes. This sudden capacity reduction sharply constrains flow on the main corridor and triggers congestion, forcing the intelligent vehicles to reroute towards the alternative routes. The penetration rate of intelligent vehicles is varied systematically from 0% to 100%, and performance is measured through average travel time, average speed, and traffic intensity on the analyzed segments, using multiple replications for robustness.

The results show that introducing vehicles with dynamic rerouting smooths traffic and prevents congestion from forming: as penetration grows, travel becomes faster and traffic intensity is redistributed across segments, relieving the most saturated links and making better use of the available capacity of alternative routes. The framework thus offers useful evidence, ahead of deployment, for designing gradual introduction strategies for this technology on interurban and metropolitan-access corridors.

Keywords: multi-agent simulation; SUMO; traffic optimization; dynamic rerouting; intelligent vehicles; peri-urban networks.

MSC (2020): 90B20, 90C35, 68T42

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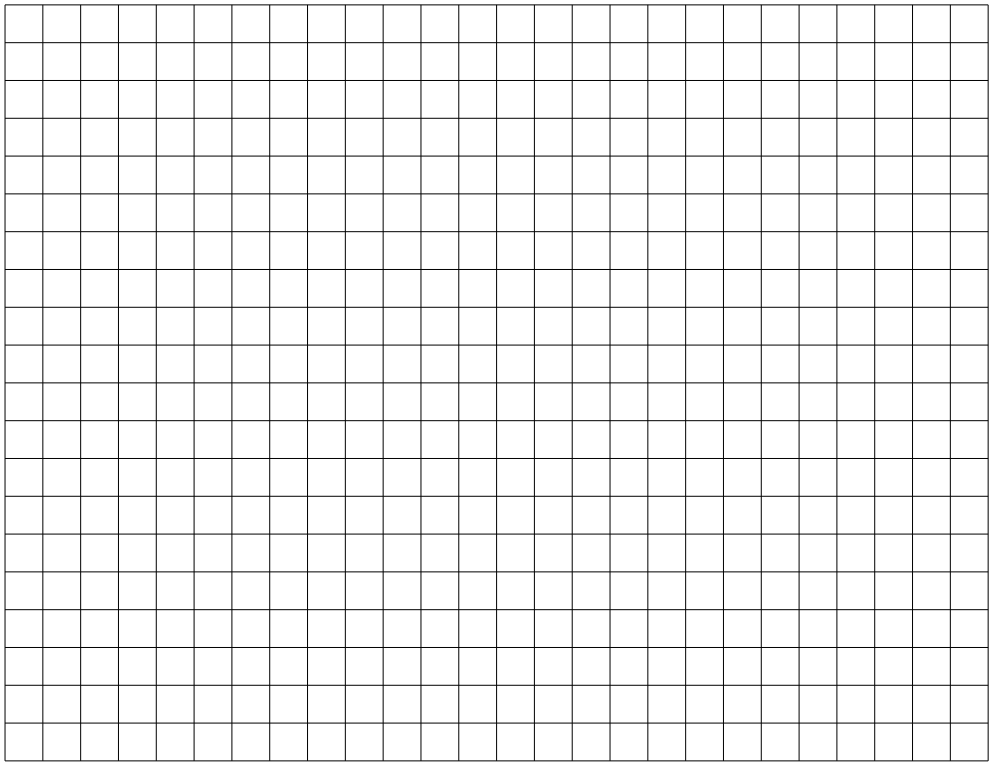

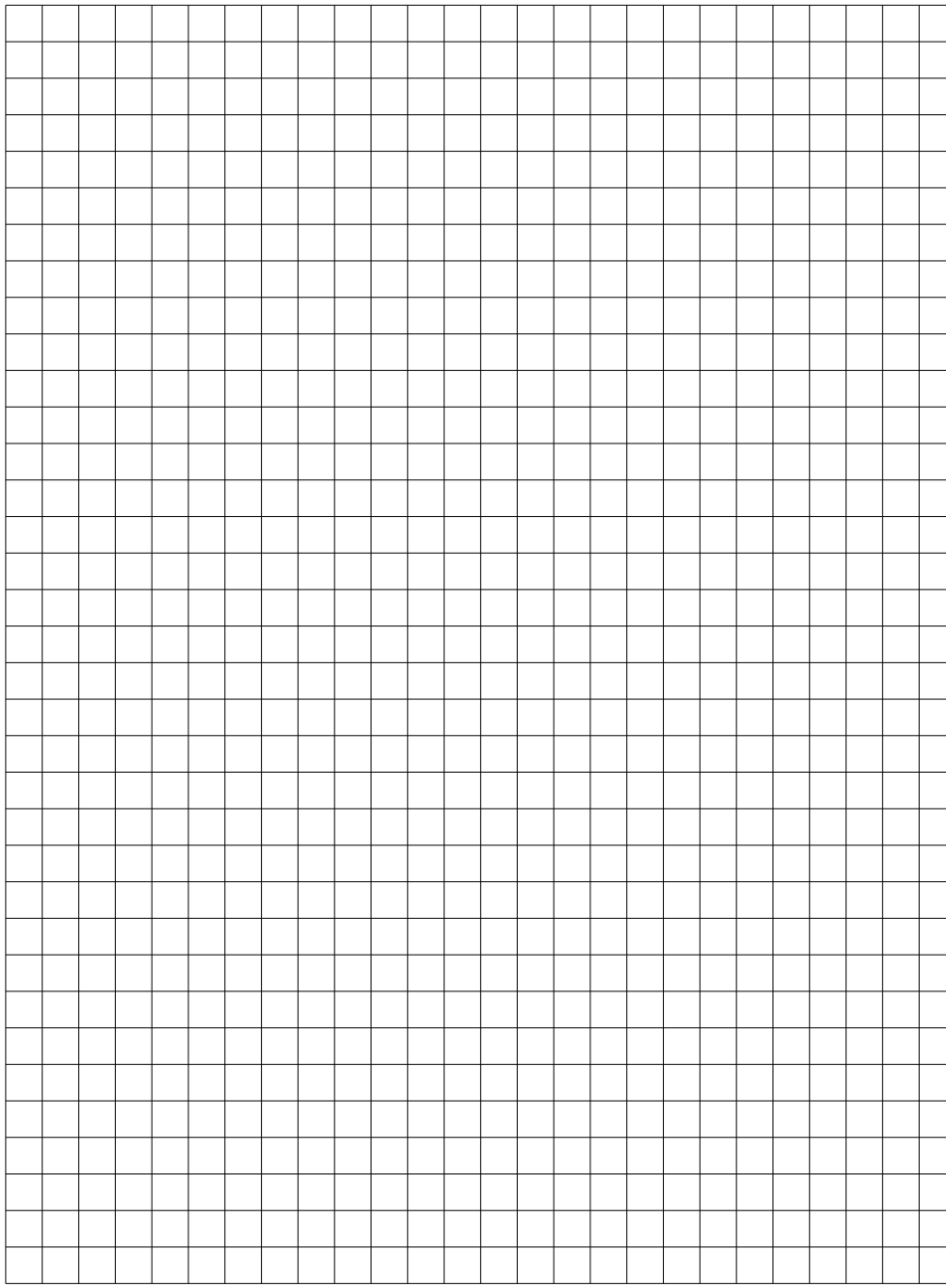

Alternating Memory in Grünwald–Letnikov Fractional Discrete-Time Systems

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Fractional discrete-time systems described by Grünwald–Letnikov-type operators with variable and alternating orders are investigated. The motivation comes from the fact that classical integer-order models do not fully capture memory effects, whereas fractional calculus provides tools for modelling processes whose current state depends on their entire previous history [3, 6, 8]. Particular attention is paid to the case in which the order function alternates between the values $\alpha \in (0, 1)$ and 1. This structure causes selected memory coefficients to vanish periodically, which can be interpreted as local reductions of memory.

The analysis is based on the Grünwald–Letnikov difference operator in convolution form, whose properties for discrete-time systems have previously been studied, for example, in [4, 5]. Due to the convolution structure, the $\mathcal{Z}$-transform can be applied, leading to an explicit expression for the operator symbol and to the characteristic mapping. This framework is then used to formulate stability conditions for the linear system in terms of the location of the eigenvalues of the system matrix with respect to the image of the unit disk. We show that the alternating-order structure modifies the stability regions. A numerical example demonstrates that stability depends not only on the spectrum of the matrix but also on the values of the fractional order. The simulations suggest the existence of a critical value, separating stable and unstable cases. The obtained results contribute to the study of variable-order systems [7, 9] and may serve as a basis for further research on numerical methods and control design for systems with structured memory [1, 2, 10–14].

MSC (2020): 39A30, 39A70

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Subdiffusion Difference Equations of Fractional Order

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This work deals with the homogeneous and non-homogeneous fractional diffusion difference equations. The fractional operators in space and time are defined in the sense of Grünwald–Letnikov. Applying results on the existence of eigenvalues and corresponding eigenfunctions of the Sturm–Liouville problem, we show that solutions to fractional diffusion difference equations exist and are given by the finite series. Obtained results are based on our recent work [1].

MSC (2020): 35K05, 26A33, 35R11

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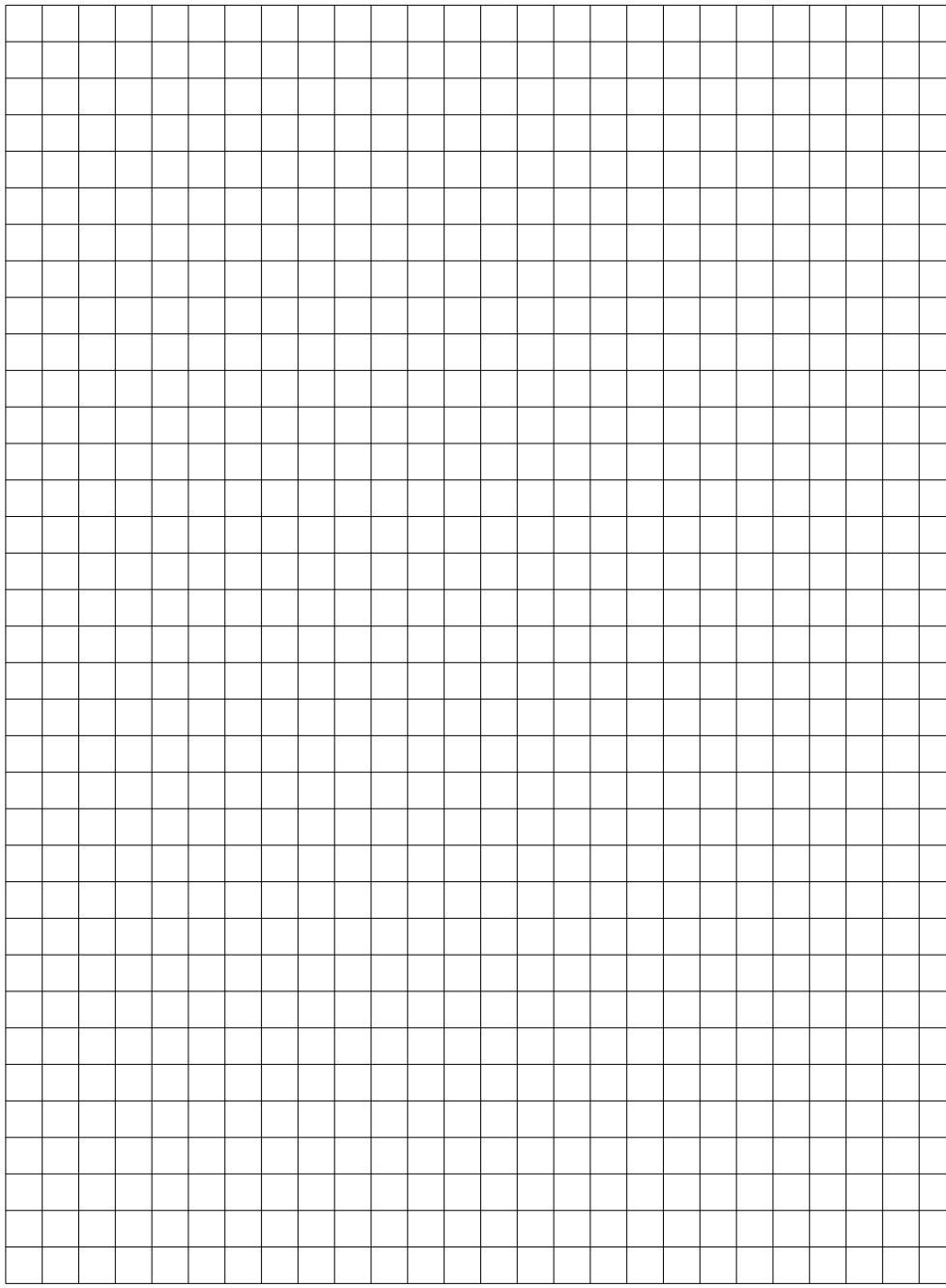

Remarks on Generalized Convexity

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We are going to present some results based on our published paper [2] concerning functions $f : D \to \mathbb{R}$ which satisfy the functional inequality:

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + \omega(x, y, t), \quad x, y \in D,\ t \in [0, 1],$$

where $\omega : D \times D \times [0, 1] \to \mathbb{R}$ is a given map and D stands for a convex subset of a real linear space. Some new results will also be presented.

MSC (2020): 26B25, 39B62, 46N10

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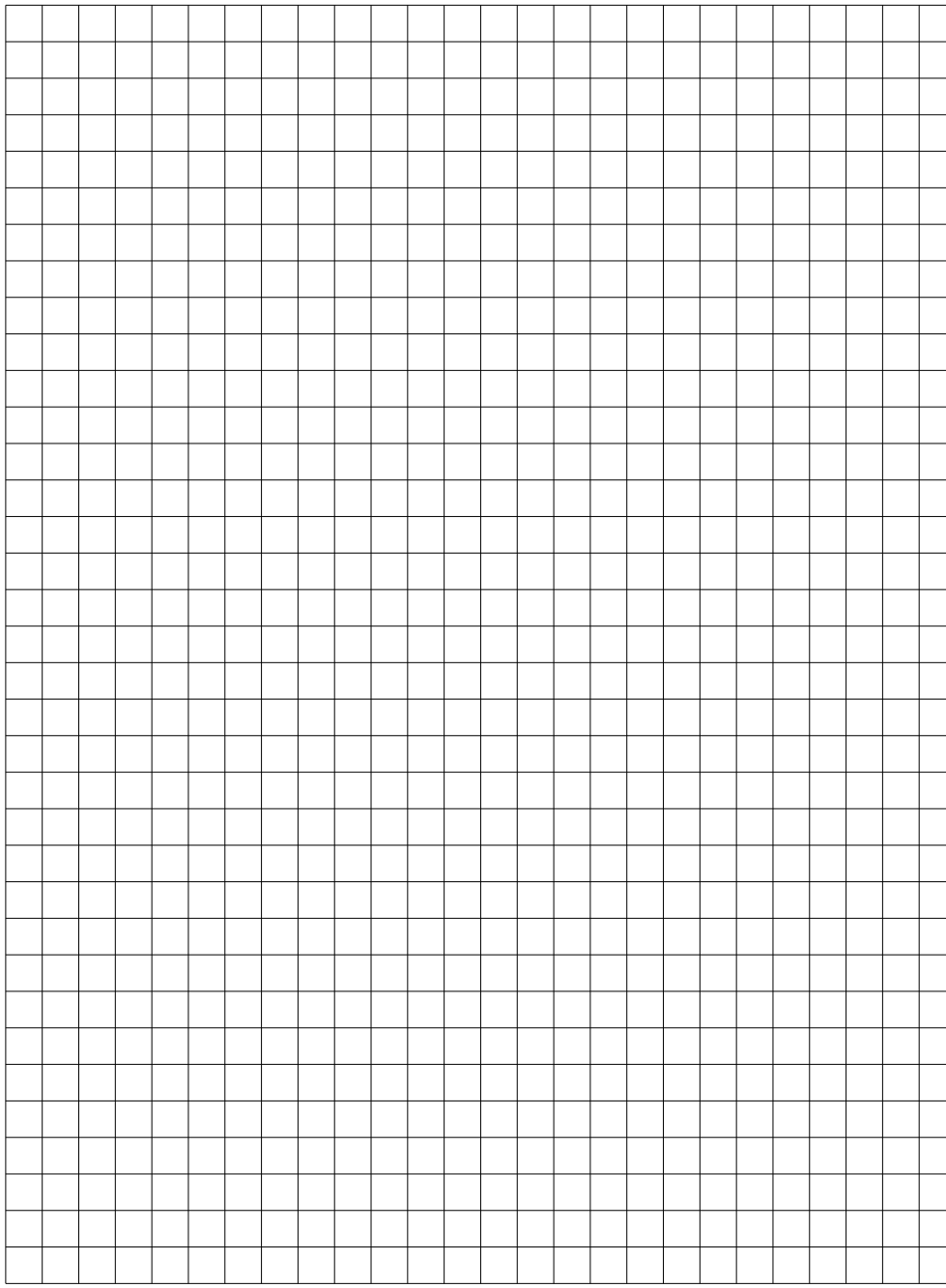

On Solving Large Scale Stochastic Dynamic Weapon Target Assignment Problems

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The problem of Weapon Target Assignments (WTA) has long attracted researchers' interest due to its practical significance and the fact that it is an NP–complete, nonlinear convex problem. Significant progress has been made in recent years regarding algorithms and solvers for this problem. Recent approaches to solving the WTA problem take into account the convexity of the objective function [2], employ problem reformulation to facilitate the efficient use of column generation [3], and propose a hybrid approach [1]—in this method, solving a relaxed convex problem in the initial stage provides a starting set of columns for the column generation process—enabling the efficient solution of static WTA problems involving more than 10 000 targets and weapons. However, practical considerations require that the WTA problem be defined with the understanding that new targets are revealed stochastically over time. This leads to the formulation of the WTA problem as a multi–step problem. This paper presents a formulation of the dynamic WTA problem and demonstrates, through numerous numerical experiments, that the problem can be effectively solved using a strategy based on a scalable Branch-Price-and-Cut algorithm.

MSC (2020): 90B80, 90C09, 90C25

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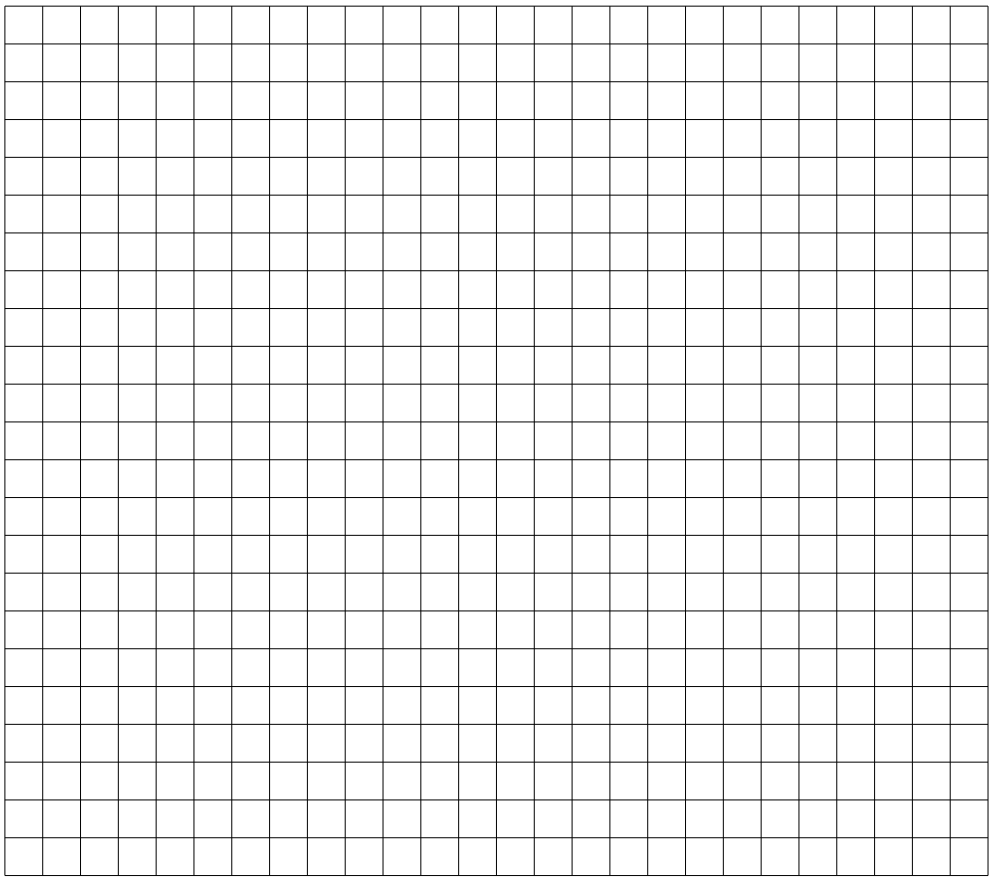

Interpretable Machine Learning for Identifying Multivariate Biochemical Signatures Associated with the Niacin Skin Test Response

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Machine learning is widely used in healthcare to analyze high-dimensional clinical data, yet small and noisy datasets often lead to unstable and unreliable model explanations. Such instability limits trust and reduces the scientific value of model interpretability. This study addresses explanation instability by introducing the Stability-Aware Feature Consensus (SAFC) framework, which aggregates permutation-based feature relevance across multiple resampled datasets. The approach prioritizes features that remain consistently informative under data perturbations, separating robust signals from noise. Evaluated on a clinical dataset of 118 subjects with over 160 variables, the results show that single-model feature importance is highly unstable despite strong predictive performance. In contrast, SAFC-based models maintain comparable accuracy while providing more stable and interpretable feature sets. These

findings emphasize the need for stability-aware methods to ensure trustworthy machine learning in healthcare.

MSC (2020): 68T05, 68T09, 92C50

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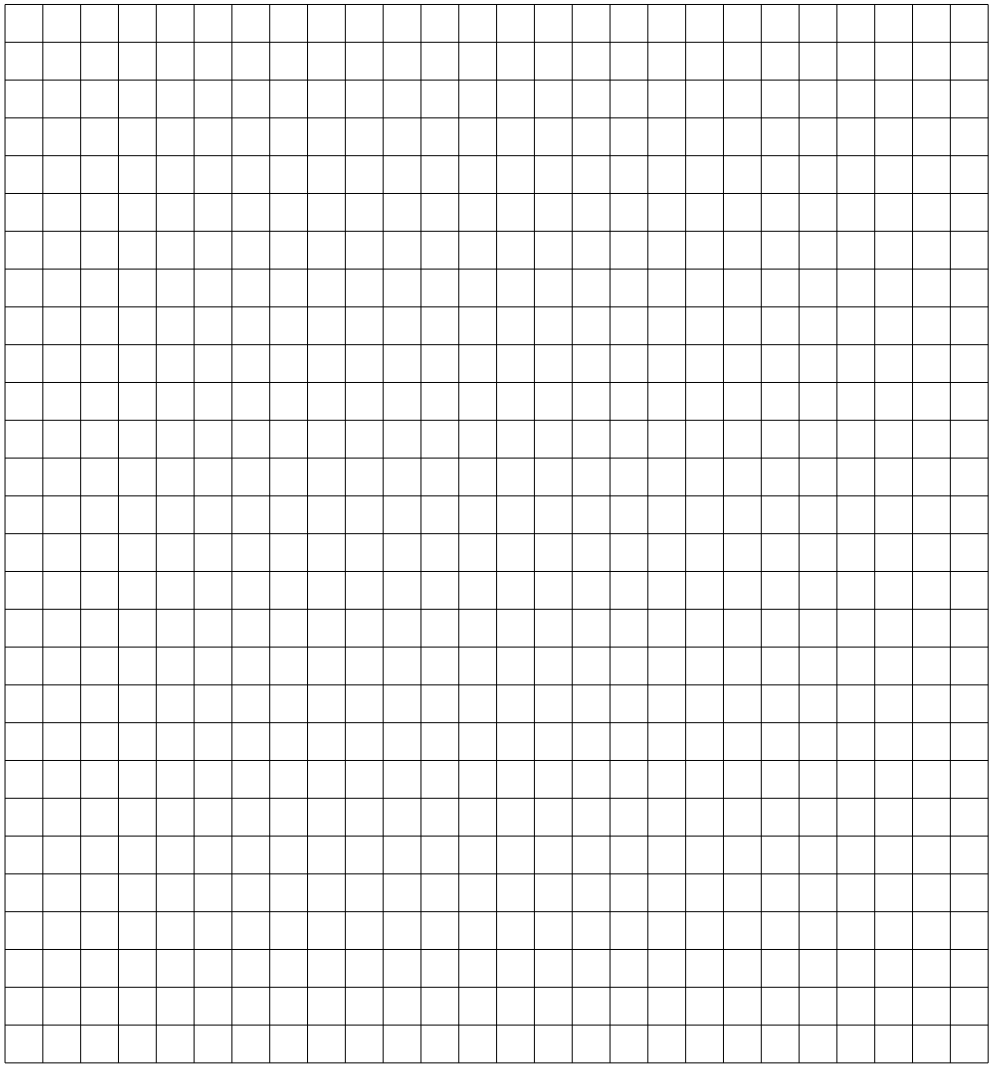

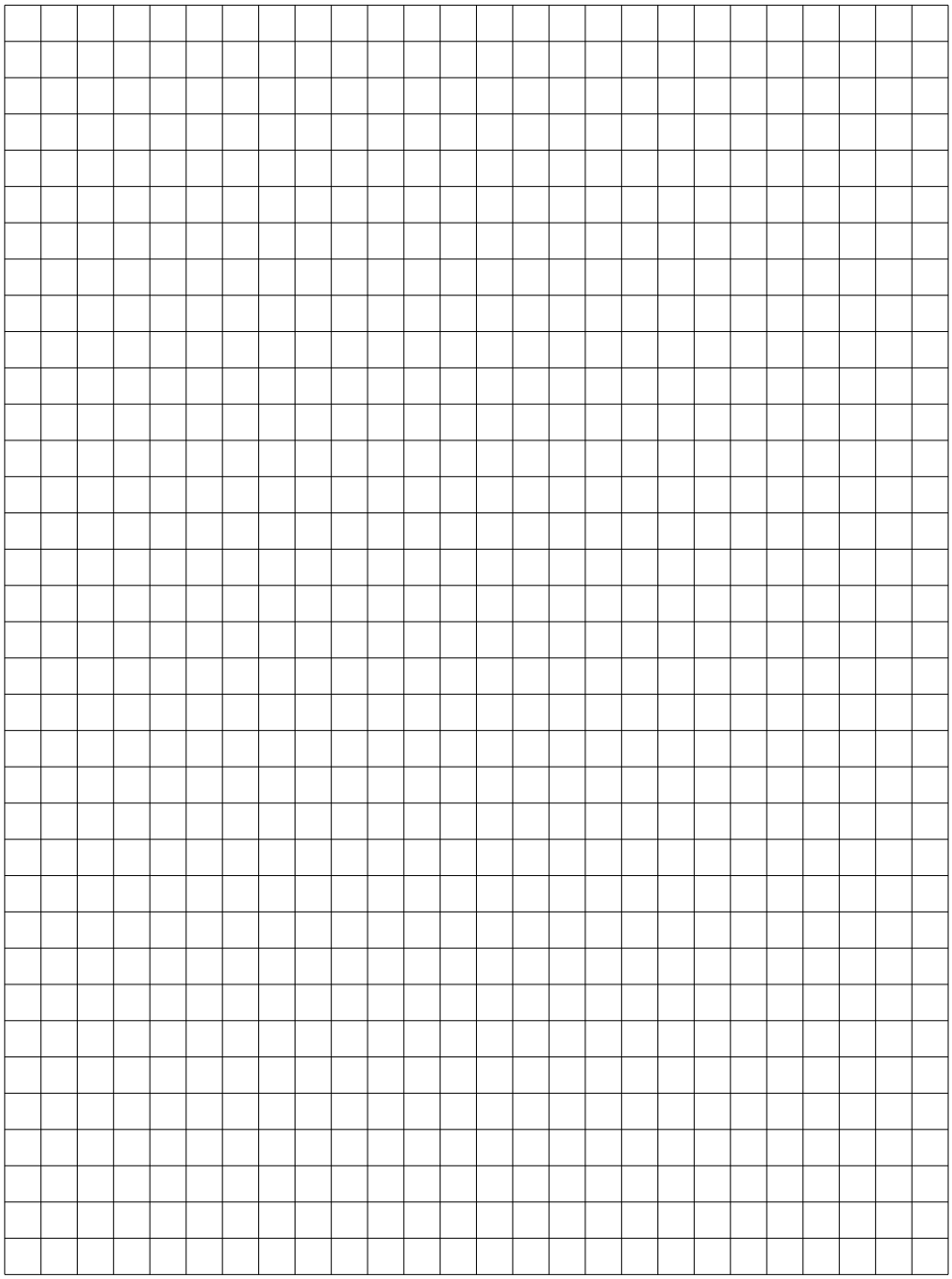

When Simpler Is Safer: Stability and Interpretability of Machine Learning Models for Clinical Biomarkers under Limited Data Conditions

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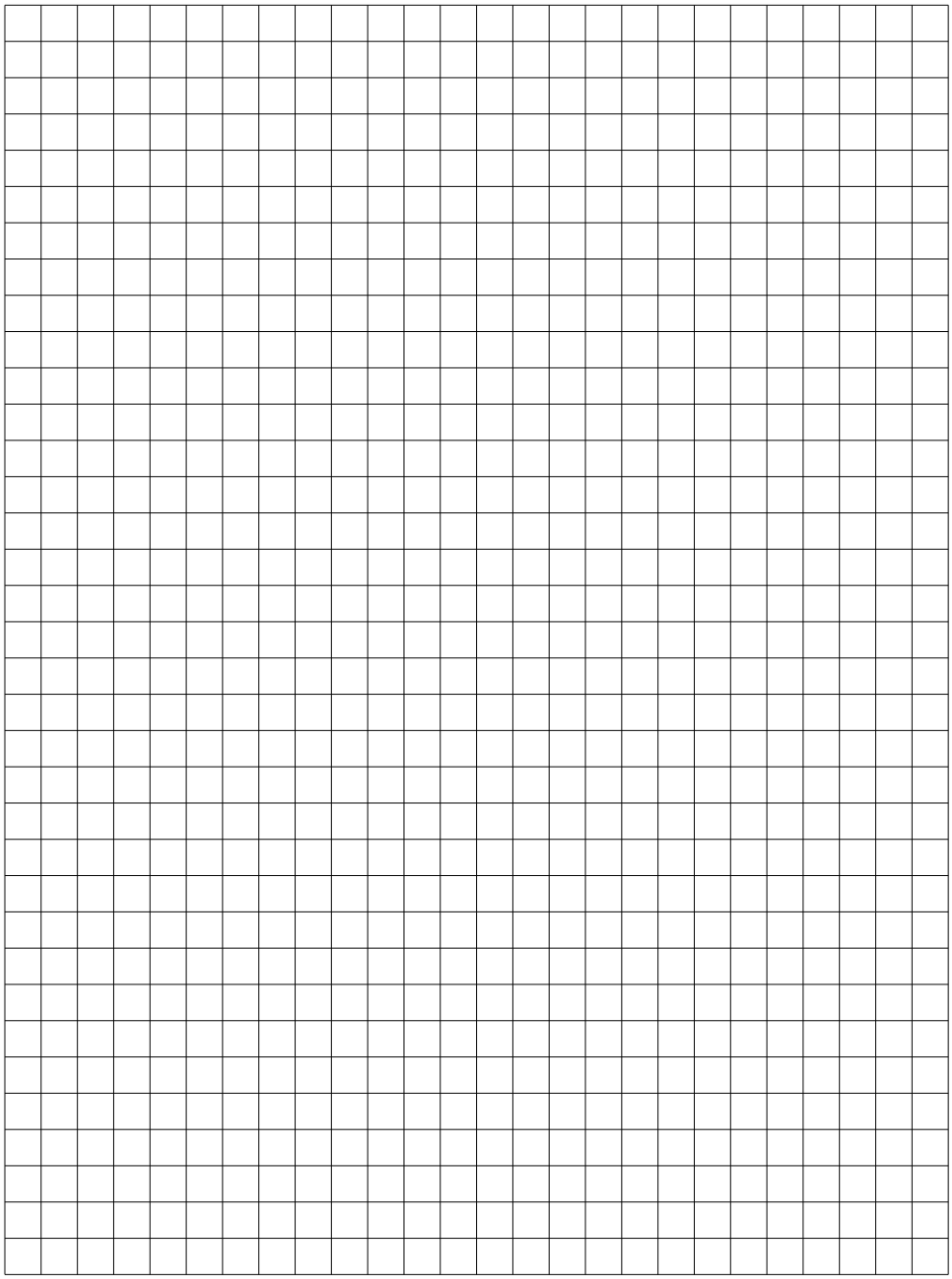
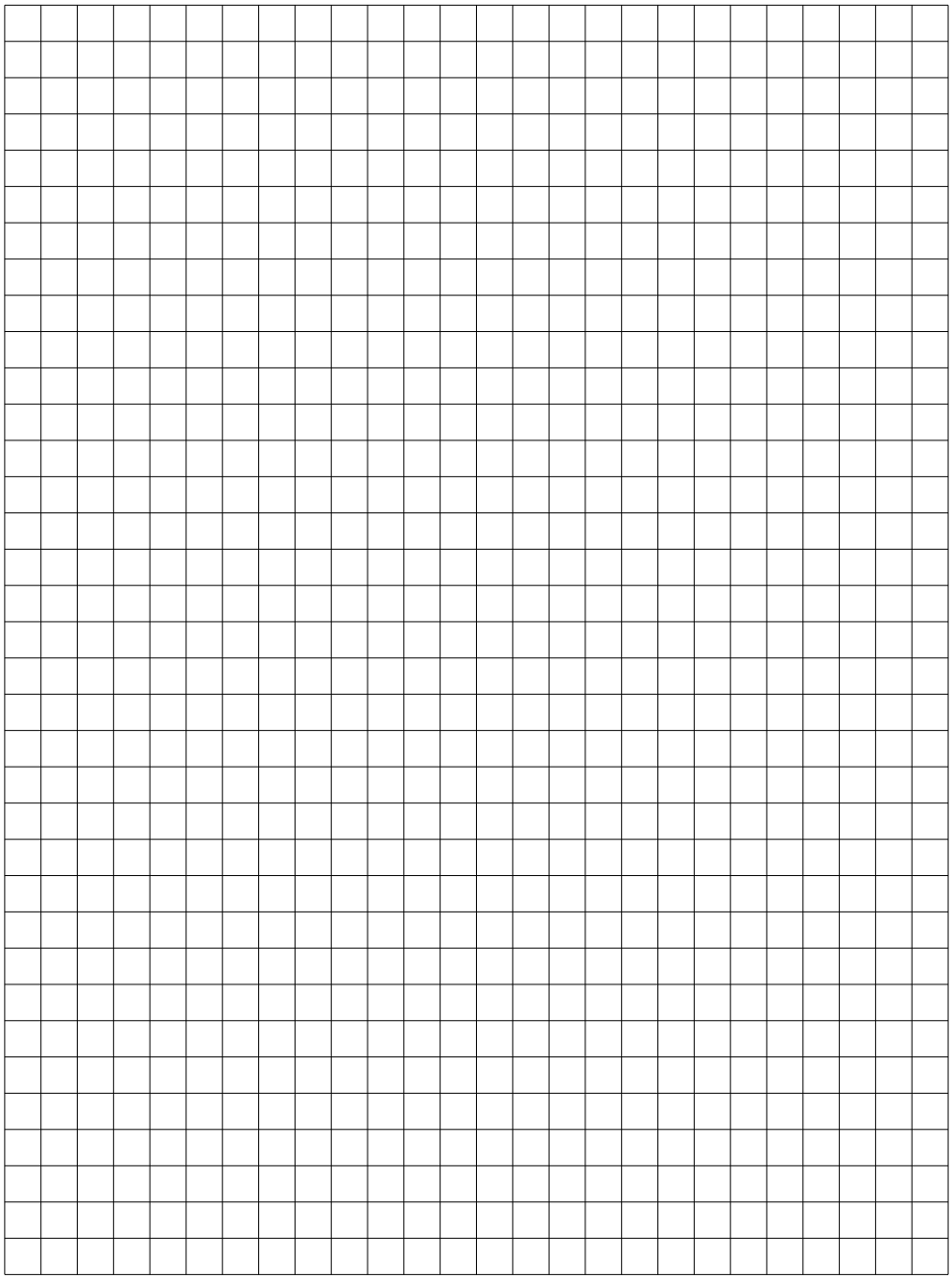
The deployment of machine learning in clinical settings requires models that are both accurate and stable under limited data conditions. This study examines the stability–performance trade-off of classical machine learning models in data-scarce scenarios. Using retinal OCT biomarkers for schizophrenia classification as a case study, we evaluate Decision Tree, Random Forest, and Gradient Boosting models across ten train–test split ratios ranging from 50:50 to 95:5, with repeated random sampling. Performance is measured using accuracy, F1-score, and AUC, with emphasis on variability across runs. Gradient Boosting achieves the highest average performance, while Random Forest shows the greatest stability under low-data conditions. Decision Trees improve with more data but exhibit high variance. The results show that models with similar peak performance can differ significantly in stability, underscoring the need for distributional evaluation rather than single-split testing. These findings support more robust model selection in clinical machine learning applications.

MSC (2020): 68T05, 68T09, 68T10, 92C55

*Presenting author.





On the Projection Problem onto a Set Defined by Quadratic Functions Lower-Level Set

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In this presentation, we investigate the problem of projecting a point onto a possibly nonconvex set defined by quadratic lower-level set constraints, i.e.,

$$\begin{aligned} &\min_{x\in\mathcal{H}} \|x-z\|^2 \\ &\text{s.t.} \quad x \in C = \{x \in \mathcal{H} : \ -A_i\|x\|^2 + \langle B_i, x\rangle + C_i \leq 0, \ i = 1,\dots,m\}, \end{aligned}$$

where $A_i \in \mathbb{R}$, $B_i \in \mathcal{H}$, $C_i \in \mathbb{R}$ for $i = 1,\dots,m$ and $\mathcal{H}$ is a Hilbert space. We examine the relationship between this general problem and its convex counterpart where $A_i = 0$ for all $i = 1,\dots,m$. Furthermore, we present sufficient conditions for global optimality and discuss necessary conditions.

MSC (2020): 90C20, 90C26

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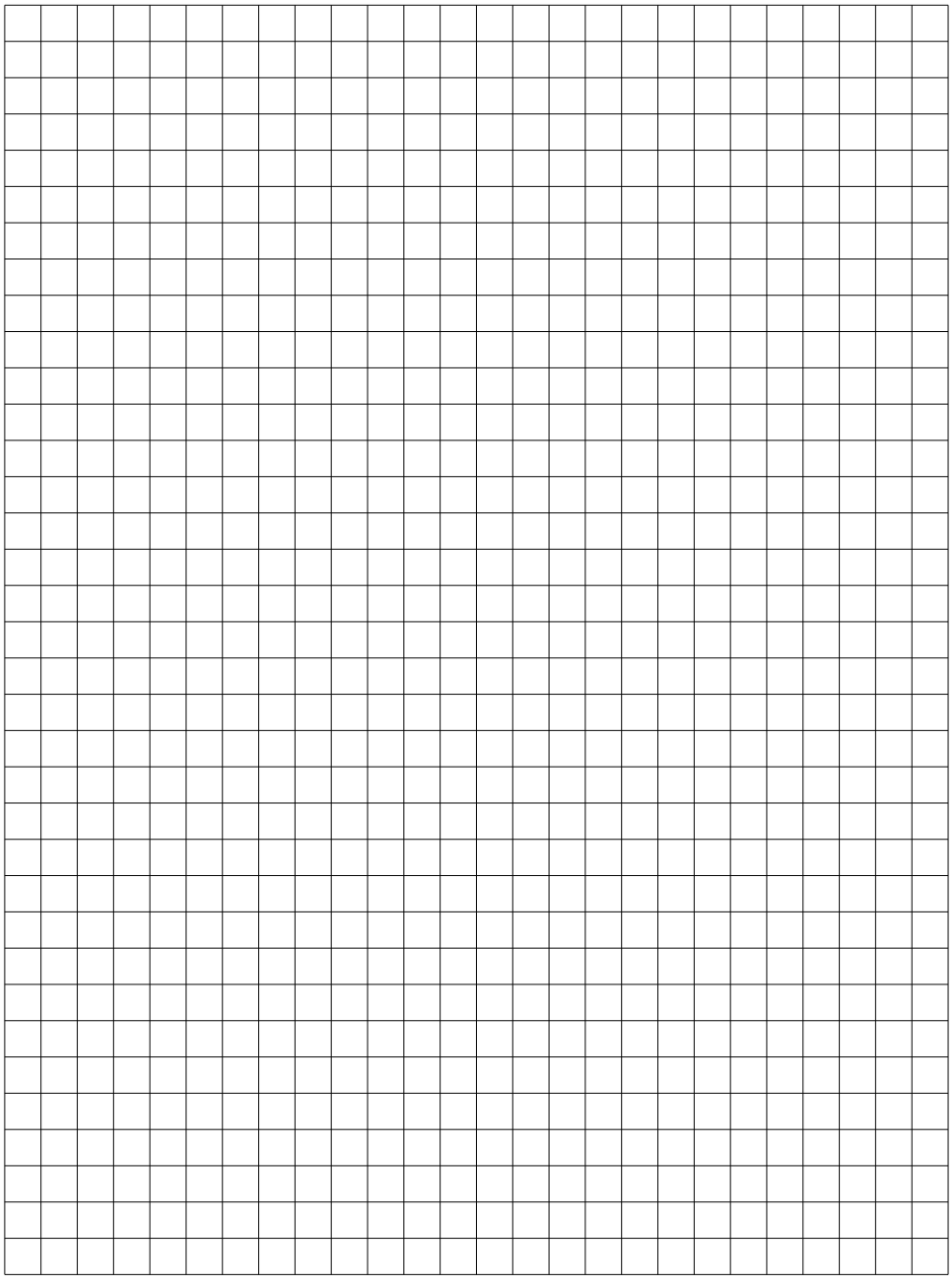

Mathematical Modeling of Dengue Dynamics: From a General Host–Vector Model to a Two–Strain Dengue Model with a Constant Recruitment

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We will discuss recently obtained results based on our latest published work [1–3]. In this study, we examine a two–strain dengue model that captures the interactions between human and mosquito populations. The proposed model incorporates the possibility of reinfection with a different strain and vertical transmission of the virus from adult mosquitoes to their offspring. We assume that the recruitment rate for susceptible larval mosquitoes is constant, which allows us to show the local stability of the disease–free equilibrium, the existence and local stability of endemic equilibrium in the case of one strain as well as the existence of two–strain stationary states. What is important, these results are obtained despite the non–constant size of mosquito population. To support our findings, we present numerical simulations with realistic dengue parameters that reflect biological phenomena.

MSC (2020): 92B05, 34A34

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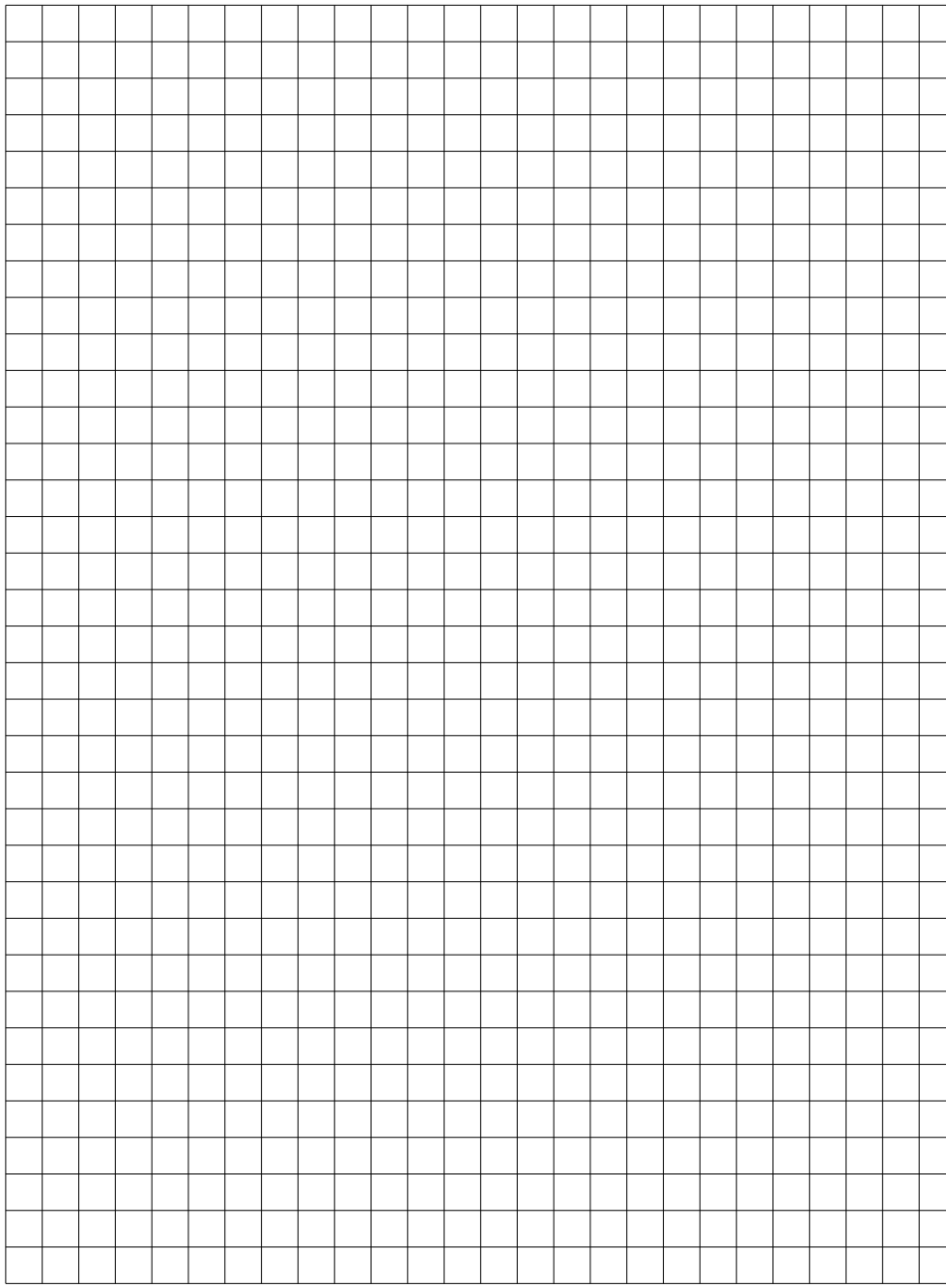

Subdifferentiability in Φ_{lsc}-Convex Analysis and Zero Duality Gap Conditions

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The talk is devoted to the study of lower semicontinuous quadratically minorized functions in Hilbert spaces within the framework of Φ_{lsc}-convexity [1]. A fundamental result establishing the existence of a global Φ_{lsc}-subgradient associated with every local Φ_{lsc}-subgradient is presented.
We discuss Φ_{lsc}-subdifferentiability properties of several classes of nonsmooth functions, including locally $C^{1,1}$ functions, twice continuously differentiable functions, prox-regular functions, and paraconvex functions. These results provide a unified subdifferential characterization of such functions in the setting of abstract convexity.

MSC (2020): 32F17, 49J52, 49K27, 49K35, 52A01

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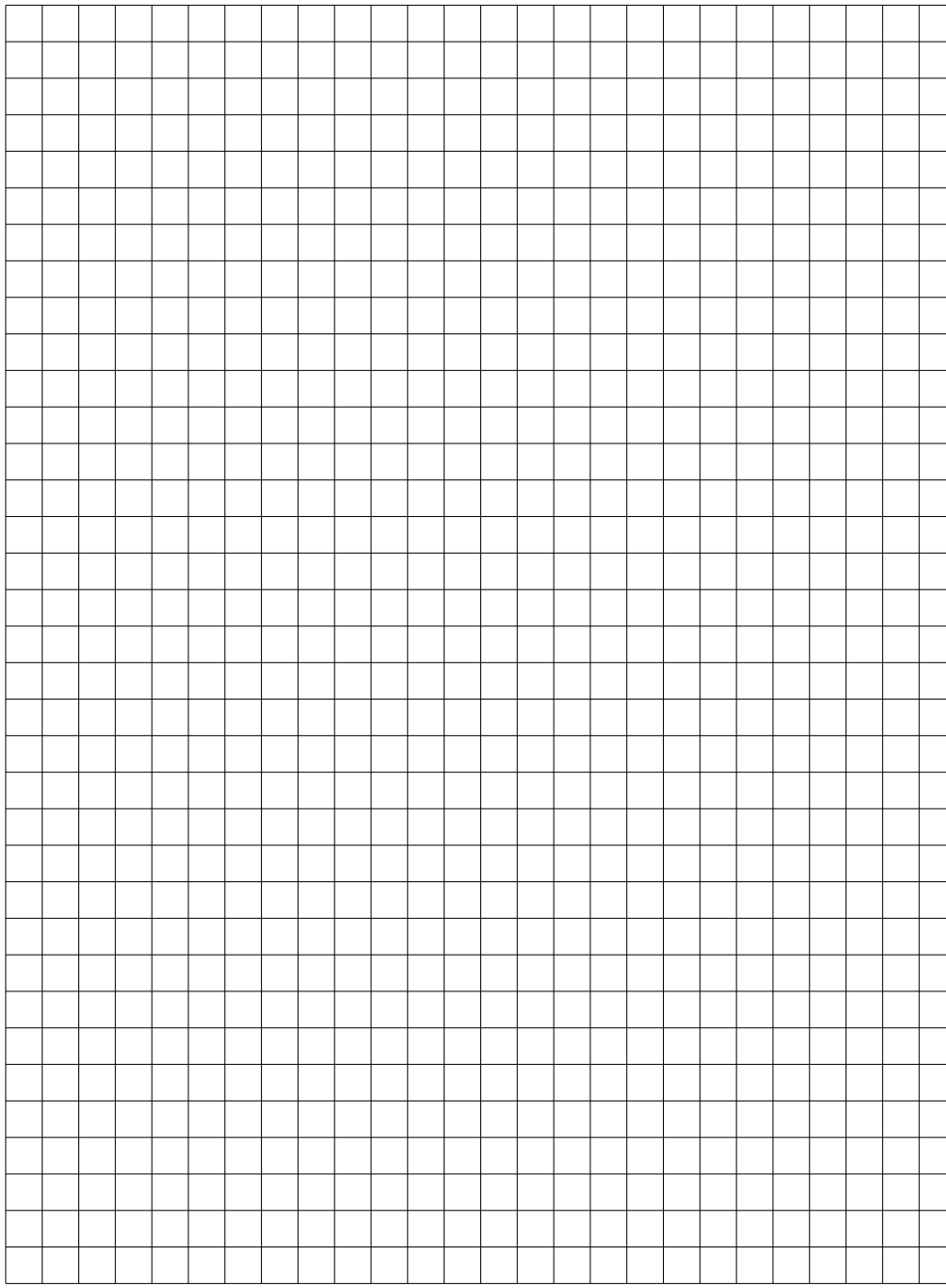

Approximation Methods for Solving the Linearized Model of a Hinged-Free Reduced Plate

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The problem of partially hinged partially free rectangular plate that aims to represent a suspension bridge subject to some external forces [1, 2] (for example the wind) is considered in order to model and simulate the unstable end behavior. Such a problem can be modeled by a plate evolution equation [3], which is nonlinear with a nonlocal stretching effect in the spanwise direction. The external forces are periodic in time and cause the vortex shedding on the structure (on the surface of the plate) and thus it may cause damage to the material [4]. Numerical study of the behavior of steady state solutions for different values of the force velocity are provided with two finite element methods of different type [5].

MSC (2020): 35J40, 35J60, 65N30, 74F10, 74S05

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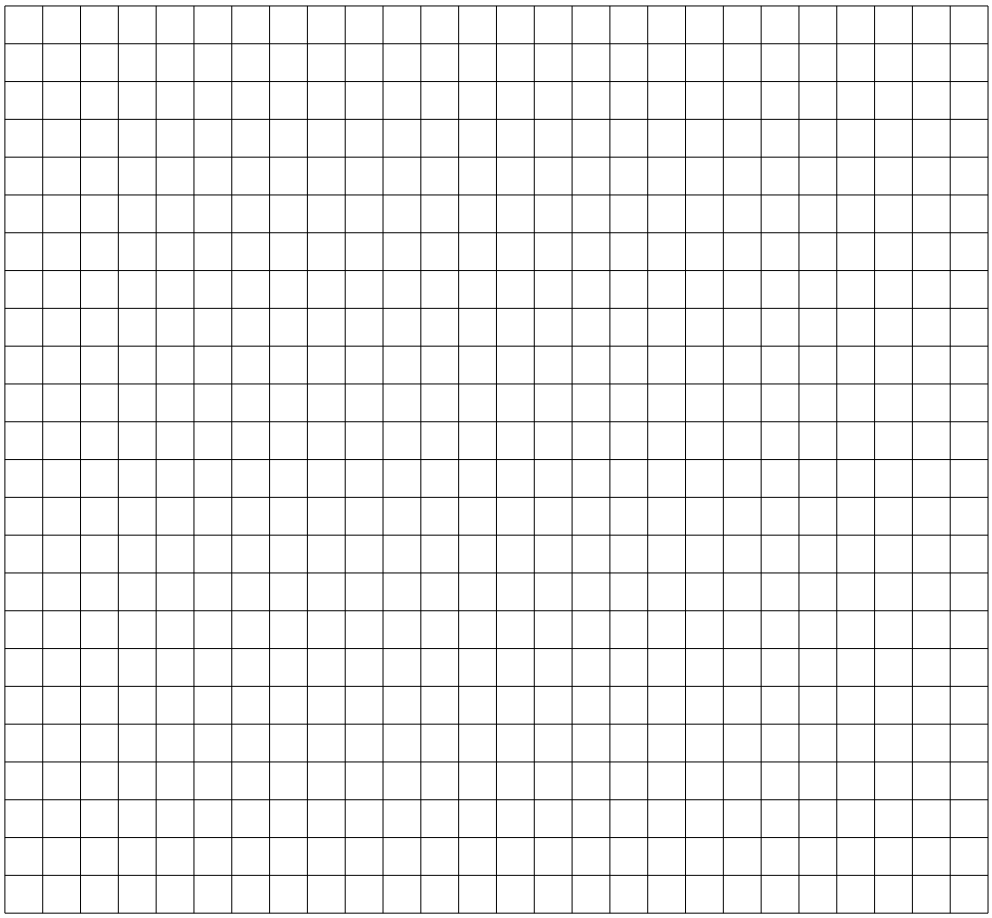

Optimized Feature Selection for Gaze-Based Classification of Pre-Inspection and Solving Phases in Speedcubing

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Skilled performance combines planning and execution, and eye movements can reveal how a person shifts between them. In competitive cube solving (speedcubing), each attempt has a pre-inspection phase, in which the solver looks at the scrambled cube without turning it, and a solving phase, the actual solve. The paper aims to find how gaze differs between these two phases. Because the phases last for very different amounts of time, we made every standard gaze measure [1] independent of duration: event counts were turned into rates per second, while per-event measures (such as mean fixation duration) and proportions already do not depend on phase length. Using 210 recordings from 85 participants, the phases with paired non-parametric tests (Holm-corrected) were compared with linear mixed models, and logistic-regression and random-forest classifiers were trained to decide whether a segment came from the pre-inspection or the solving phase, validated so that no participant appeared in both training and test. The eyes moved just as often in both phases, but the way they moved differed: during solving, fixations became longer and dwell time increased, while saccades became smaller and slower, reflecting a shift from broad, exploratory scanning towards focused inspection [2], [3]. From gaze alone, the classifiers told the two phases apart with ROC-AUC ≈ 0.84, and the result held even when only saccade measures were used (AUC ≈ 0.69–0.75), showing that it is not merely an artifact of the cube's motion (smooth pursuit). Exhaustive subset selection identified three duration-independent features (fixation rate, saccade rate, and mean fixation duration) that classify the two phases as accurately as the full set (AUC ≈ 0.84) and are selected in every fold. Planning and execution can therefore be reliably told apart from gaze, which could support gaze-based assessment of visuomotor skill.

MSC (2020): 62H30, 68T05, 62P15

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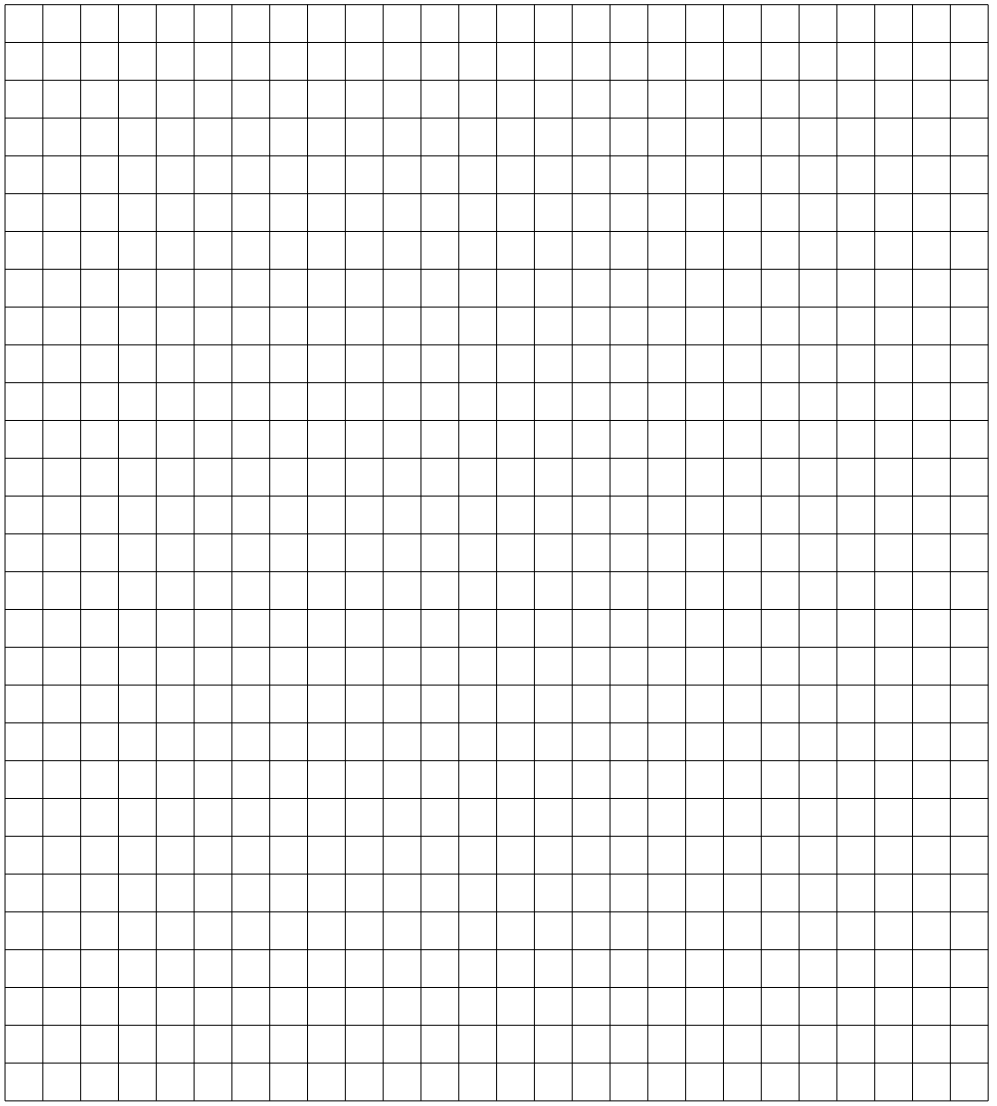

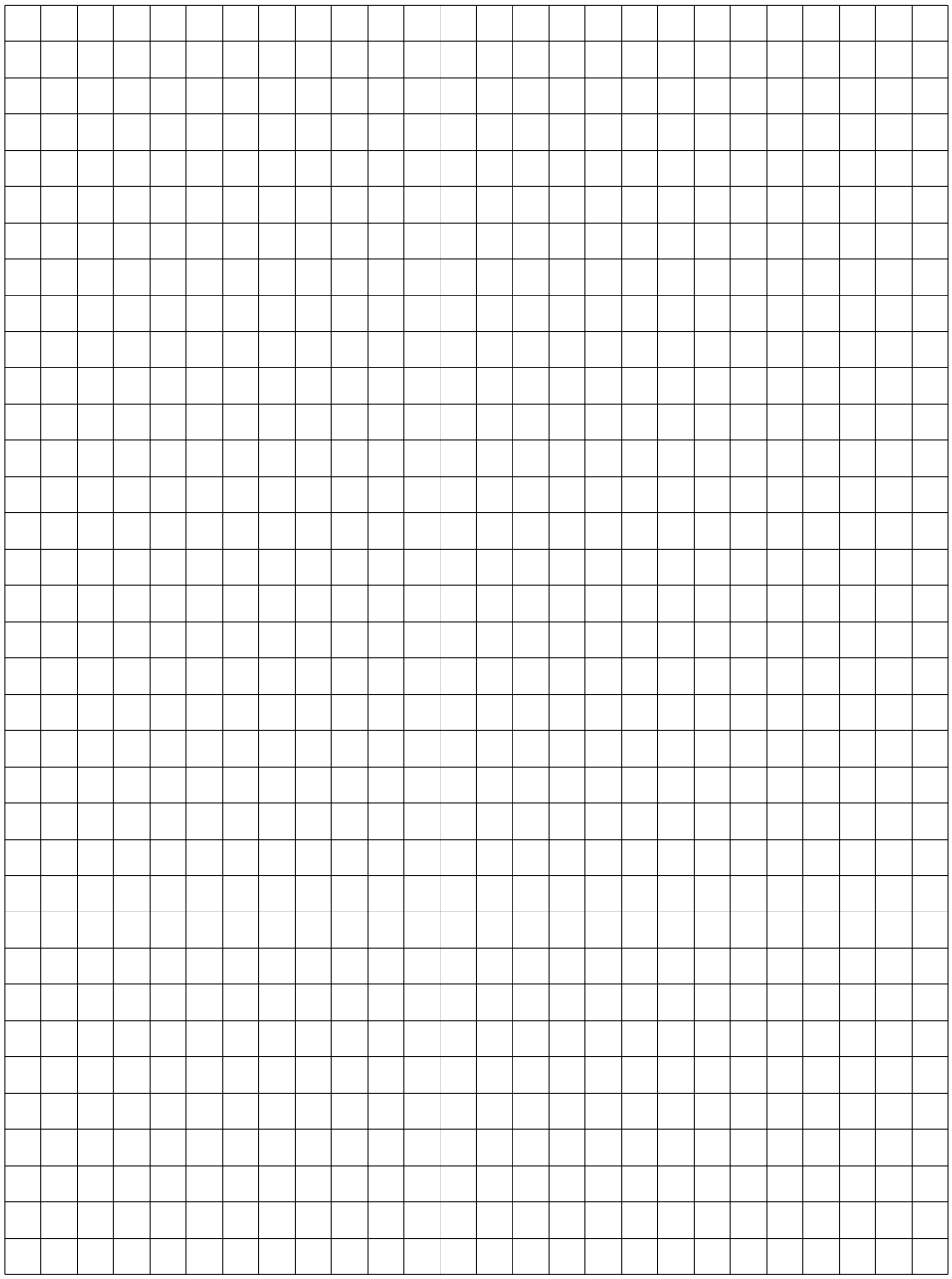

Proximity Characterization in the Setting of Abstract Convexity

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We consider proximity operator as a selector of the subgradient in the context of abstract convexity and characterize its properties in term of minimization problems. We also investigate the continuity and differentiability of proximity operator in the case of paraconvex and weakly convex functions. Supporting examples are given in each section. This work is an extension of [1] and [2].

MSC (2020): 90C26, 37B05

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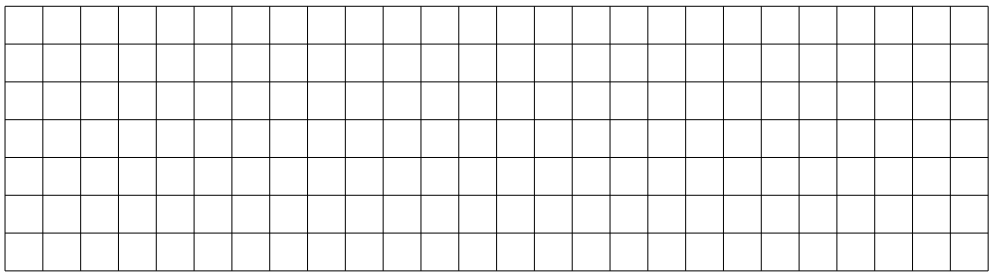

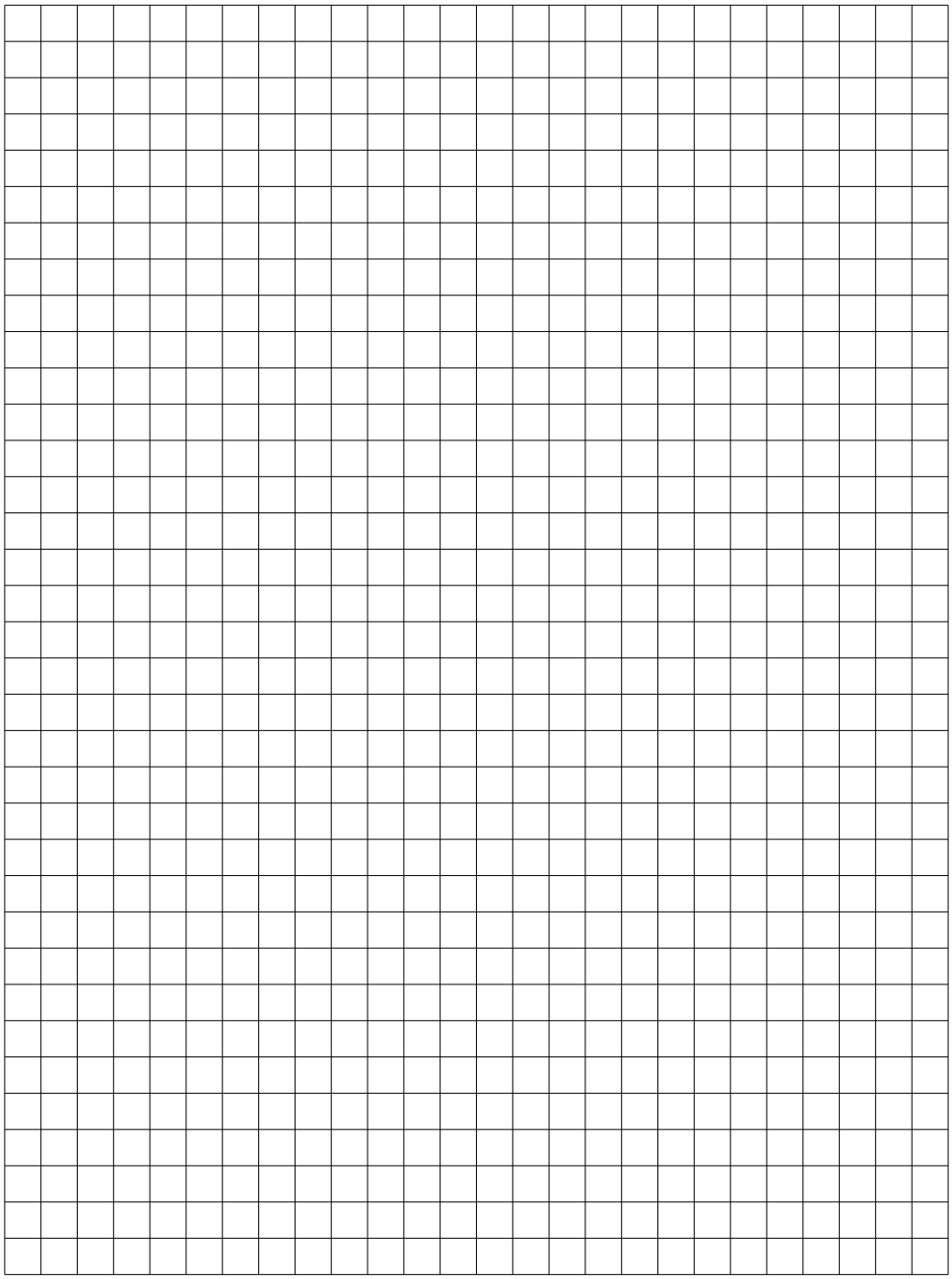

Performance Optimization of Winch Launches in Gliding

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Gliding enables sustained flight without an onboard engine. A higher release altitude generally increases the time and distance available for finding updrafts, making launch performance operationally important. Gliders are commonly launched either by aerotow or by a ground-based winch. Whereas the release altitude in aerotowing can be selected rather freely, a winch launch is limited by the initial rope length and by the fact that the winch can only reel the rope in. This raises the question of how the system should be controlled to obtain the best possible state at release.

During a winch launch, the glider is connected to a fixed-position winch by a rope and accelerated by reeling it in. The winch operator controls the motor torque, which simultaneously influences the towing speed and the traction force. At the same time, the pilot adjusts the glider's pitch and thus its angle of attack. Due to the rope dynamics, winch operation and aircraft motion cannot be studied independently. We therefore model the complete launch as a coupled, time-dependent optimal control problem. Earlier work used simplified cable constraints [1], while later the system was simulated [2].

The winch model distinguishes between the operator's command and the resulting drum motion. A first-order actuator model incorporates the delayed reaction of the motor torque to the input. This torque accelerates the drum against rope tension and mechanical friction. The reeling speed determines the decreasing unstretched length of the free rope.

To resolve the distributed rope dynamics, the rope is divided into elastic segments connected by point masses, with its endpoints attached to the winch and the glider. Each segment obeys a tension-only linear elastic material law. As the rope is reeled in, the unstretched length of every segment decreases with

the remaining free rope length. Every internal rope node follows Newton's law under the tensions of its two adjacent segments, its weight, and damping. The ground reaction is described via complementarity constraints. Accurate prediction of the resulting rope tensions is also relevant to launch safety [3].

The glider state comprises position and velocity, pitch angle, and pitch rate. Airspeed is computed relative to the wind. The equations of motion combine lift, drag, and gravity with the traction force transmitted by the final rope segment. A pitching-moment model describes stability around a trim angle, including pitch damping. The glider's ground contact is again modeled by complementary constraints. The primary objective maximizes the glider's total mechanical energy at the terminal point. Weak penalties on different variables suppress nonphysical oscillations and select numerically stable trajectories.

For numerical solution, we discretize the differential equations by an implicit Euler scheme, yielding a nonconvex nonlinear program. It is implemented in AMPL and solved with CONOPT [4]. Before solving the dynamic problem, a separate static nonlinear program determines a mechanically consistent resting configuration from the endpoint positions, elasticity, gravity, and ground contact. This solution is then used as a starting point for the numerical solver.

After parameter estimation and validation, the model will be used to quantify performance and safety trade-offs. The resulting numerical experiments are intended to show which physical effects materially change an optimal launch strategy and how winch operation interacts with feasible glider dynamics under realistic constraints and different weather conditions.

MSC (2020): 90C30, 90C90

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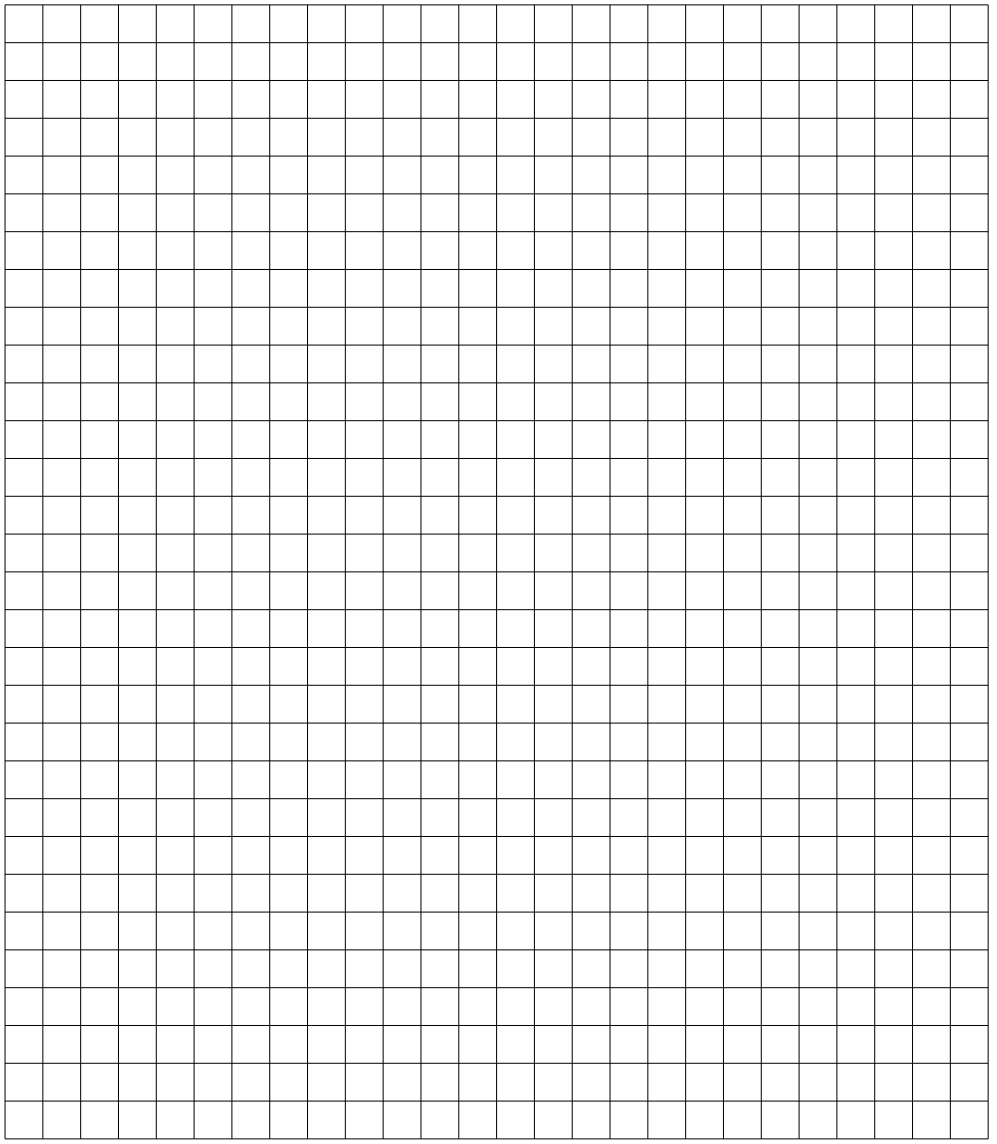

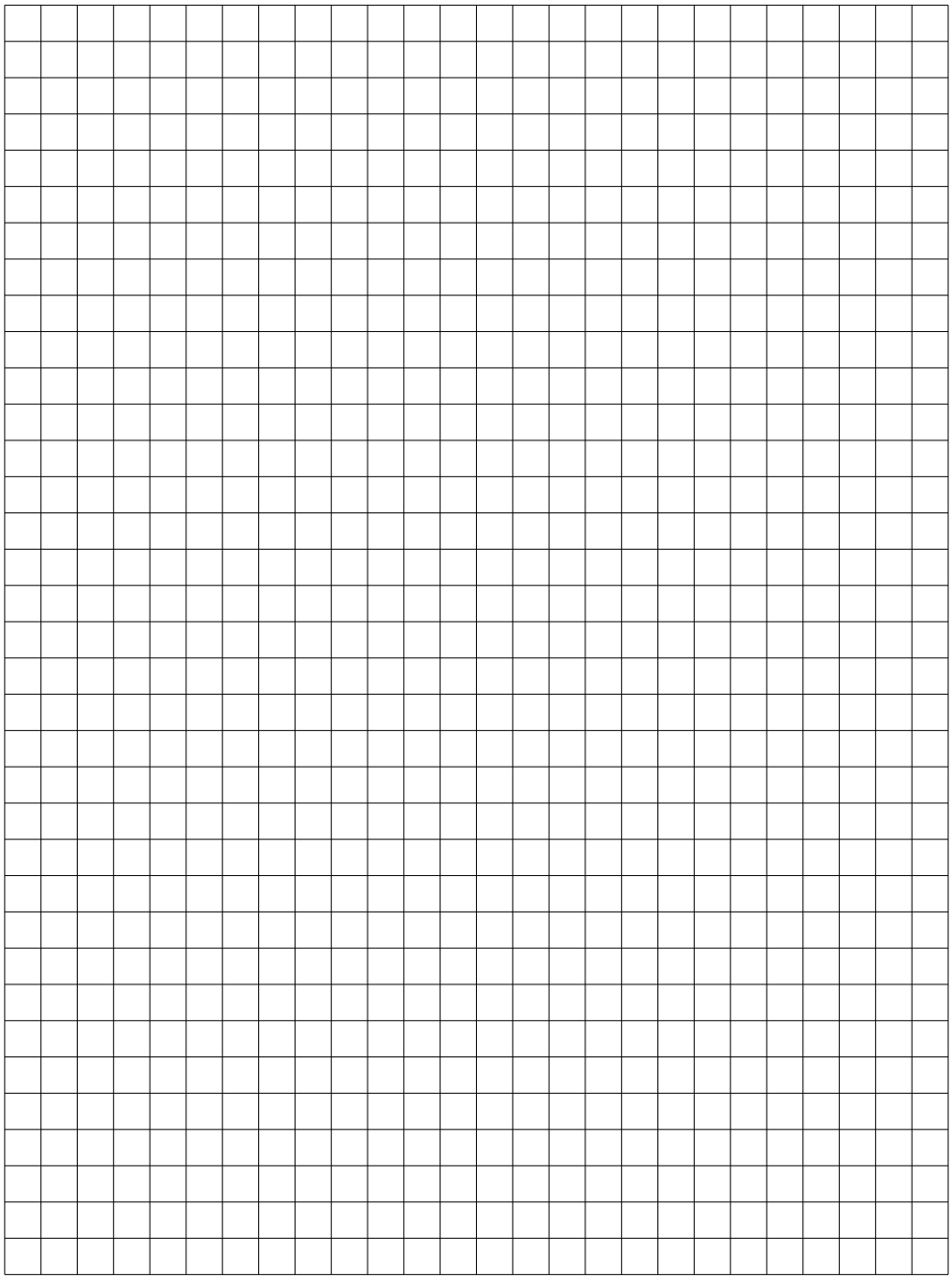

Stability Analysis of Fractional Discrete-Time Systems with Alternating Orders

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Fractional calculus provides a natural framework for modeling dynamical systems with memory. In the discrete-time setting, fractional-order systems and their stability properties have been extensively studied; see, e.g., [2, 6]. Variable order fractional models have attracted increasing attention, as they allow the memory characteristics of a system to vary with time [1, 3–5].

In this work, the stability of a discrete-time fractional system with alternating orders α and β defined by the Grünwald–Letnikov operator is investigated. The periodic variation of the fractional order gives rise to a structured memory pattern in which selected past states do not contribute to the current dynamics. Such memory gaps distinguish the considered model from standard constant-order fractional systems and may substantially affect its stability properties.

An explicit recursive representation of the system is derived, making the underlying memory mechanism transparent and providing a convenient framework for stability analysis. Stability regions are investigated for selected pairs of alternating orders α and β, including the fractional–integer case, in which one of the alternating orders is equal to one. The results illustrate how the interaction between the two fractional orders and the resulting memory gaps affects the stability characteristics of the system and provides a basis for further stability analysis of more general variable-order fractional models.

MSC (2020): 93D20, 39A30

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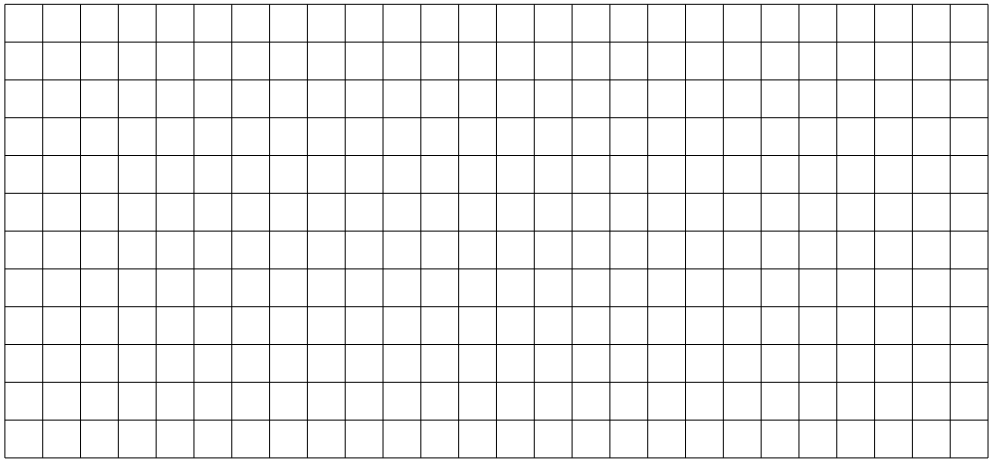

Analysis and Optimization of Methods for Solving the Discrete Logarithm Problem

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This lecture is devoted to the discrete logarithm problem, which is one of the fundamental challenges in number theory and modern cryptography. The first part presents the definition of the discrete logarithm problem and discusses its significance in public-key cryptographic systems, particularly in the Diffie–Hellman key exchange protocol. Subsequently, three selected methods for solving this problem are examined: the brute-force method, Pollard's rho algorithm, and the Pohlig–Hellman algorithm. For each method, the underlying principles, computational complexity, and applicable conditions are described. Particular attention is given to a comparative analysis of the efficiency of these algorithms in terms of computational cost and practical applicability. The aim of this paper is to evaluate the possibilities for optimizing the process of solving the discrete logarithm problem and to identify the method that offers the highest efficiency depending on the characteristics of the problem under consideration.

MSC (2020): 11G20, 94A60, 14H52, 12E20

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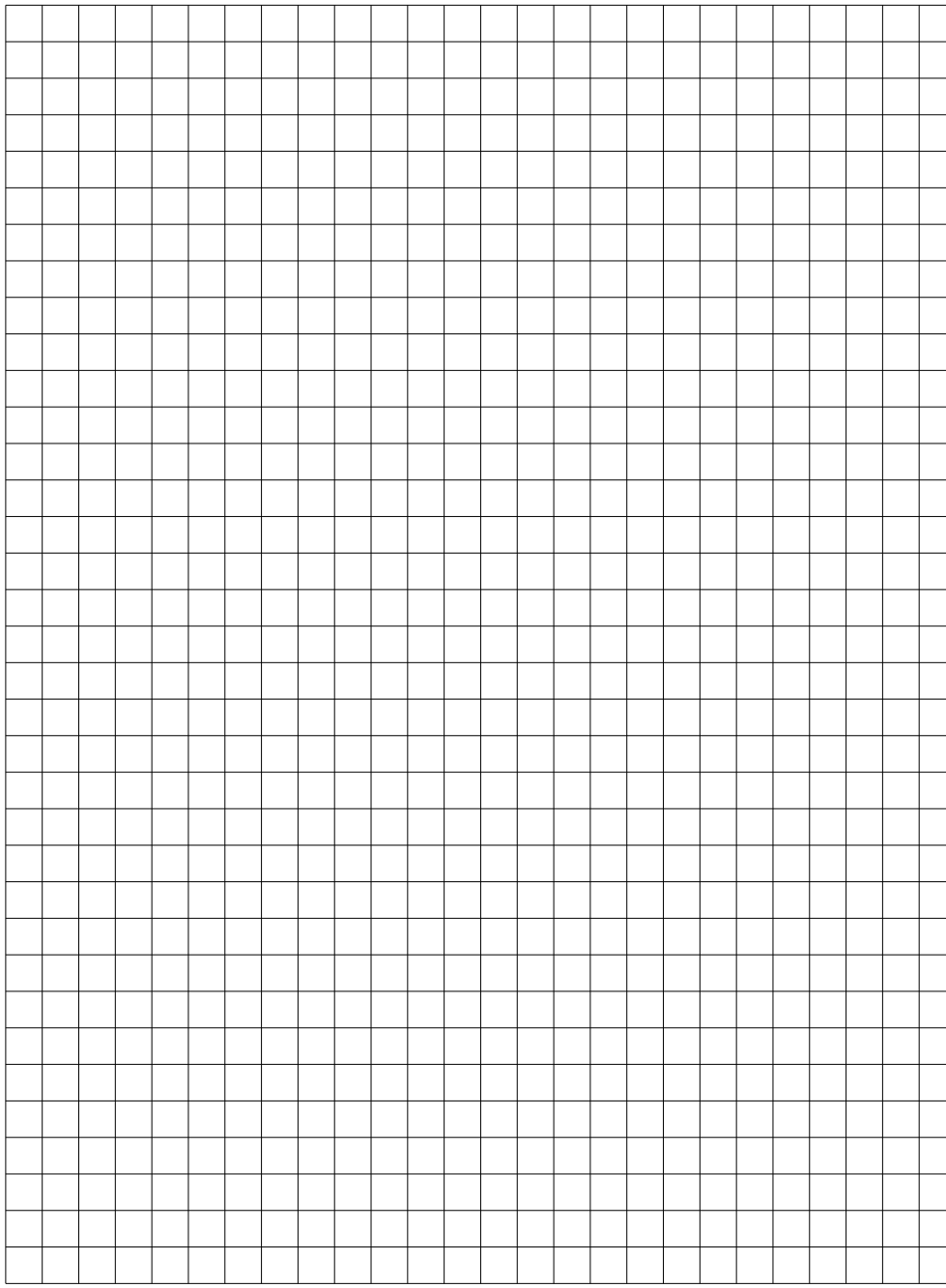

Global-in-Time Well-Posedness of Two-Level Knowledge Growth Mean Field Games in Economics

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In this talk, we study the simplest non-trivial endogenous growth model which is a Poisson-driven mean field game model for knowledge growth with optimal allocation of time and two knowledge levels; this model can certainly find immediate use in the Economics of time allocation for apprenticeship learning and professional training. From the standard dynamic programming approach, its solution can be characterized by a forward-backward system of nonlinear ordinary differential equations, and we establish the global-in-time well-posedness of this solution under the strict concavity of the learning function and some mild conditions. To see the sharpness of our proposed conditions, we further provide an illustration with a linear learning function in which multiple solutions are still feasible. To the best of our knowledge, our present work provides the first coherent and rigorous proof on well-posedness of this representative example of the knowledge model, and our study also paves the way for the complete resolution of the long lasting problems under generic settings. This talk is based on a joint work with Dantong Chu (CUHK), Tak Kwong Wong (Shenzhen University), and Sheung Chi Phillip Yam (CUHK).

MSC (2020): 49N80, 49L20, 35F21

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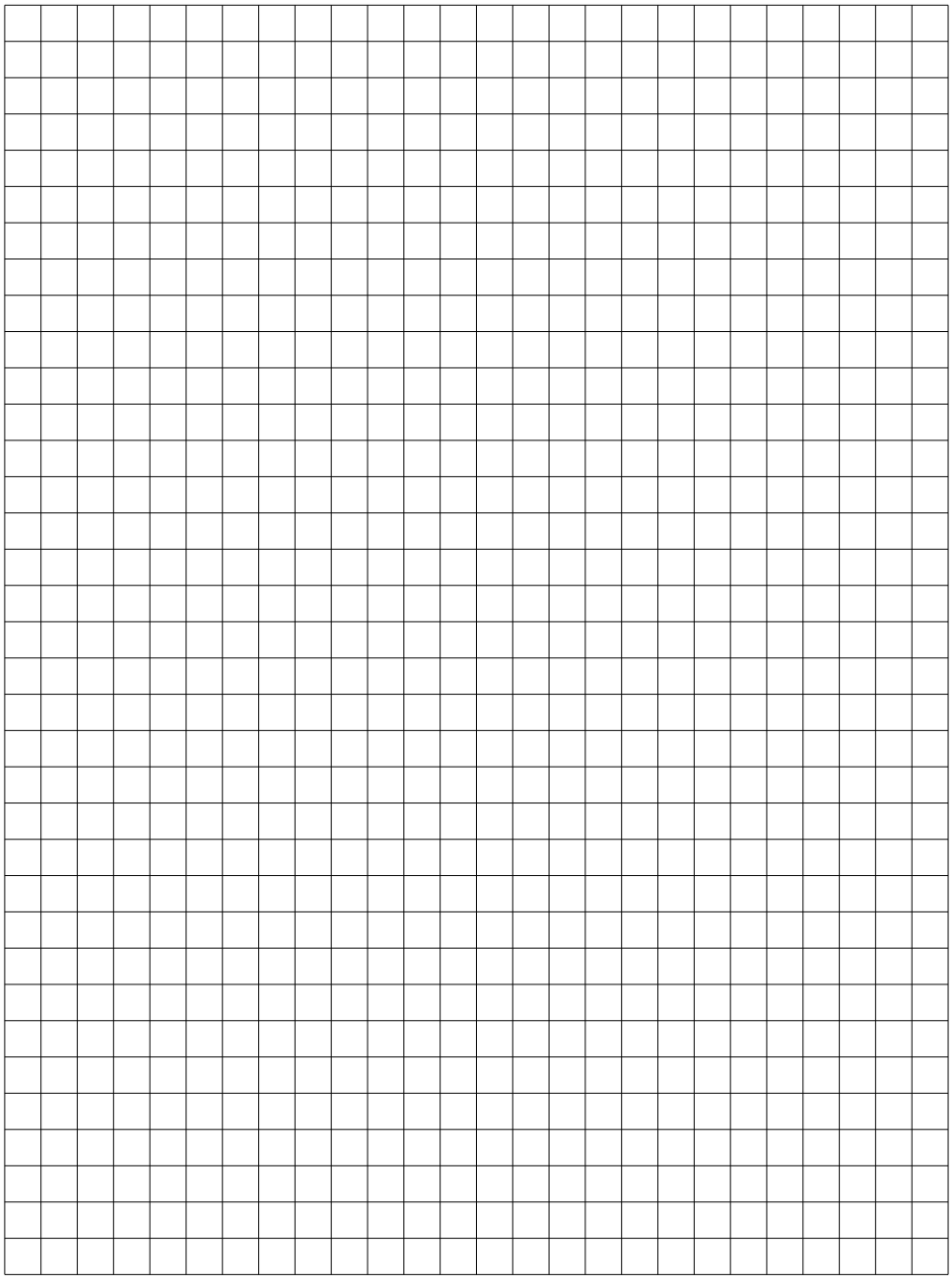

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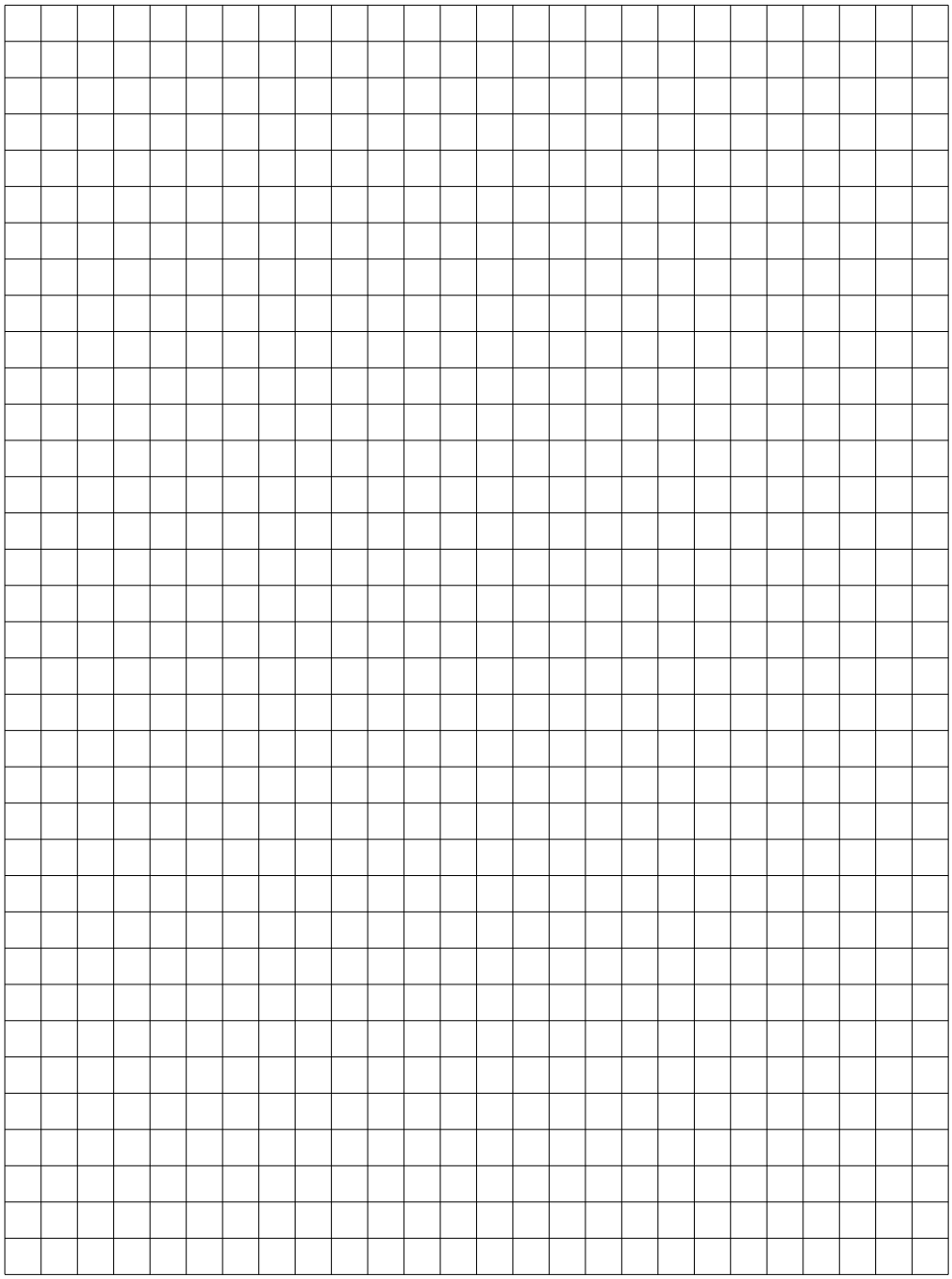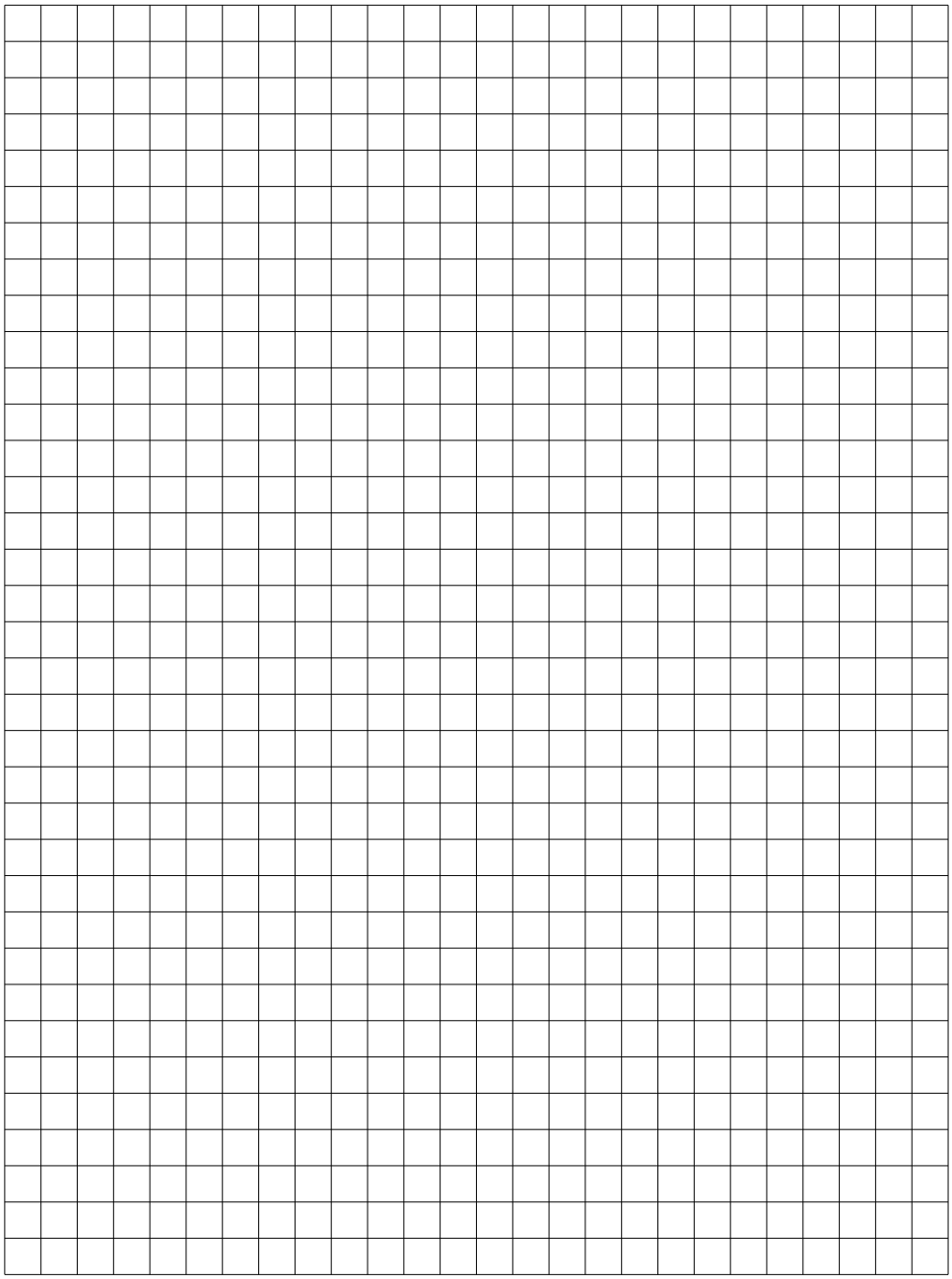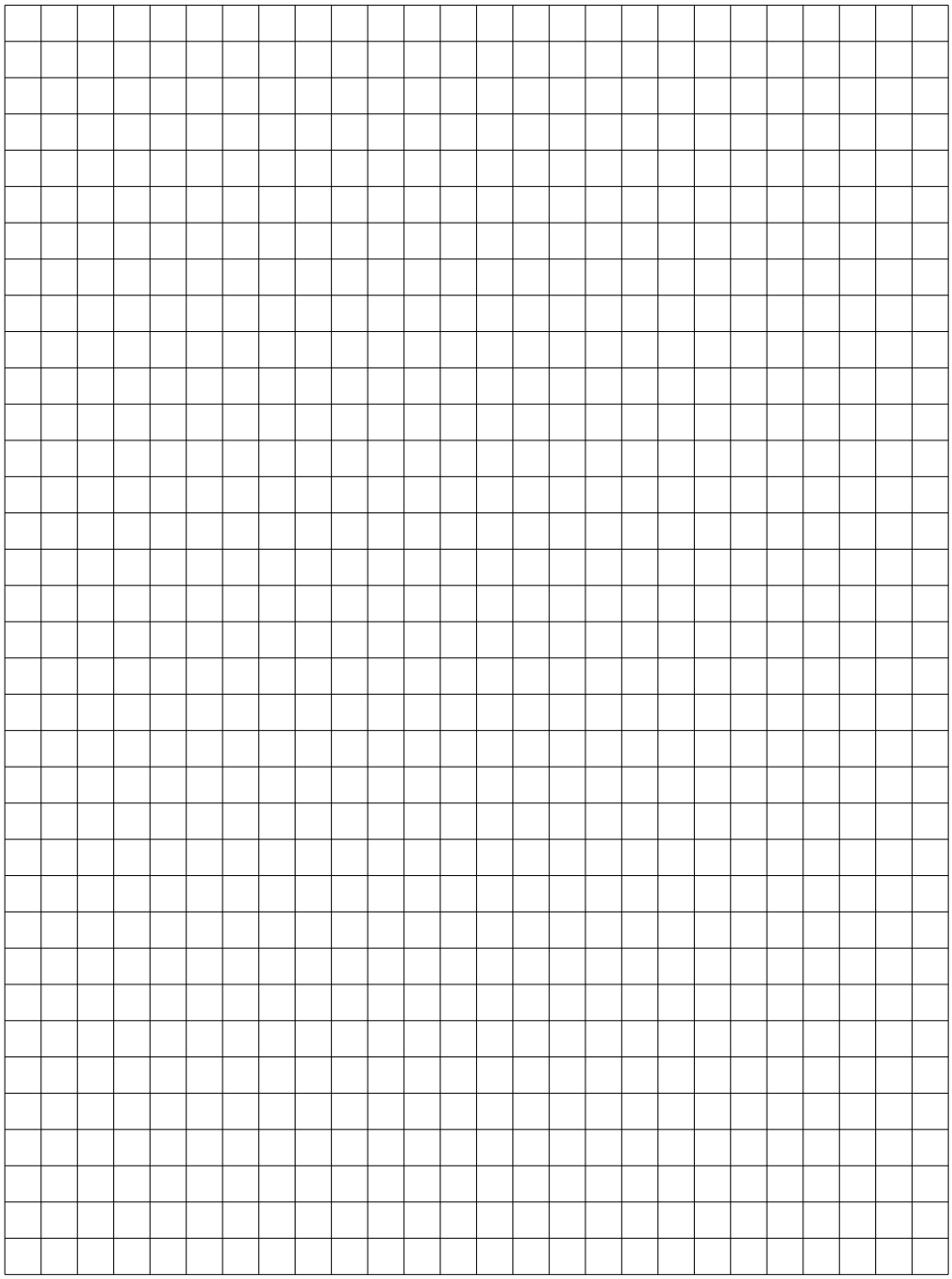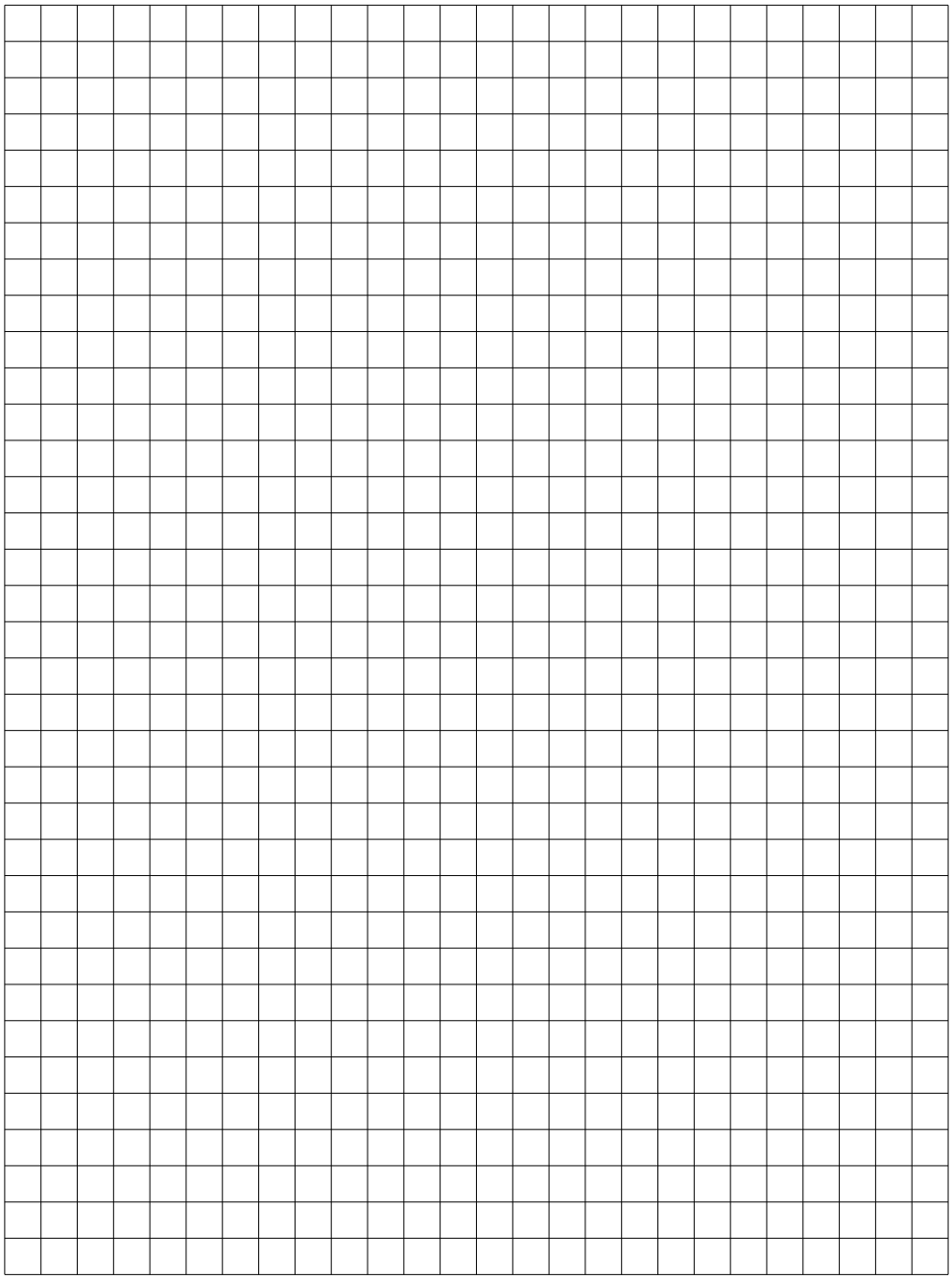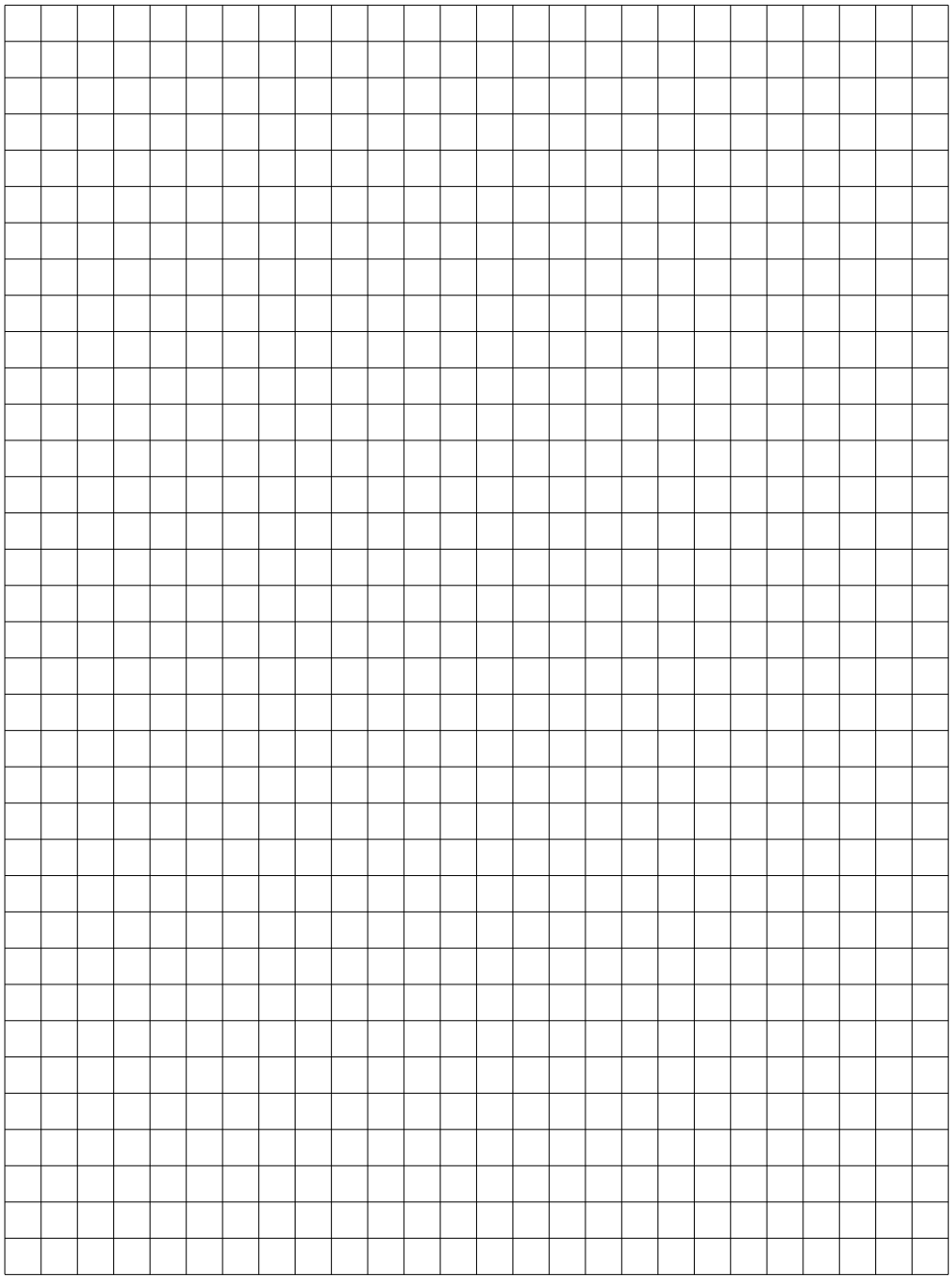
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